Planck Scale Physics and Newton's Ultimate Object Conjecture

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According to Newton, the ultimate building blocks of matter are hard frictionless spheres. This conjecture is here analyzed under different assumptions, which are:

1. The ultimate objects of matter are frictionless positive and negative Planck mass particles obeying nonrelativistic Newtonian mechanics.
2. The Planck mass particles interact with the Planck force c^4/G (c velocity of light, G Newton's constant) locally within a Planck length r_p, with the positive Planck mass particles exerting a repulsive and the negative Planck mass particles an attractive force.
3. Space if filled with an equal number of positive and negative Planck mass particles, whereby in the average each Planck length volume r_p^3 occupies one Planck mass particle.

Making these three assumptions we derive:

1. Nonrelativistic quantum mechanics as an approximation with departures from this approximation suppressed by the Planck length.
2. Lorentz invariance as a dynamic symmetry for energies small compared to the Plank energy.
3. The operator field equation for the previously proposed Planck aether model of a unified theory of elementary particles.

In contrast to theories in which the ultimate objects are strings at the Planck scale, the alternative theory proposed here does not need a higher dimensional space, but rather can be formulated in $3+1$ dimensions.

1. Introduction

In his "Optics", Newton makes the conjecture that the ultimate building blocks of matter are hard frictionless spheres. Newton's mechanical system has its perfect counterpart in Einstein's gravitational vacuum field equations, because in both, the dubious concept of long-range forces is eliminated by kinematic constraints, in Newton's mechanical system by boundary conditions on the surface of the colliding spheres, and in Einstein's field equations by a noneuclidean space-time metric. Present attempts to arrive at a more fundamental understanding of elementary particle physics have in common to go to ever larger groups (leading to an ever growing number of elementary particles.) Without exception all these attempts assume that quantum mechanics and the theory of relativity provide a frame for an ultimately correct description of nature. In pursuing this trend to the extreme, the hypothesis has been made that the ultimate building blocks of matter are Planck scale strings. This hypothesis is unsatisfactory for two reasons: First, such a string theory can be formulated only in at least 10 dimensions (the case of supersymmetric strings), rather than in the four space-time dimensions of physics. Second, the strings have very large groups (of several 100), implying the existence of a much larger number of elementary particles than actually observed.

Making a few assumptions, similar but departing from those made by Newton (who assumed that the ultimate building blocks of matter are hard frictionless spheres) we derive quantum mechanics (with a spectrum of elementary particles, greatly resembling the spectrum of the known elementary particles) and Lorentz invariance as a dynamic symmetry for energies small compared to the Planck energy. These assumptions are:

1. The ultimate objects of matter are frictionless positive and negative Planck mass particles obeying nonrelativistic Newtonian mechanics.
2. The Planck mass particles interact with the Planck force c^4/G (c velocity of light, G Newton's constant) locally within a Planck length r_p, with the positive Planck mass particles exerting a repulsive and the negative Planck mass particles an attractive force.

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3. Space if filled with an equal number of positive and negative Planck mass particles, whereby in the average each Planck length volume r_p^3 occupies one Planck mass particle.

From the two Planck relations

$$G m_p^2 = \hbar c, \quad m_p r_p c = \hbar . \tag{1.1}$$

($h = 2\pi\hbar$ Planck's constant) Planck's mass, length and time are obtained:

$$\begin{aligned} m_p &= \sqrt{\hbar c/G} \simeq 10^{-5}\ \text{g}, \\ r_p &= \sqrt{\hbar G/c^3} \simeq 10^{-33}\ \text{cm}, \\ t_p &= \sqrt{\hbar G/c^5} \simeq 10^{-44}\ \text{sec}. \end{aligned} \tag{1.2}$$

Expressed in terms of these units, the Planck force is

$$F_p = c^4/G \simeq 10^{50}\ \text{dyn}. \tag{1.3}$$

Unlike m_p, r_p and t_p, F_p does not depend on $\hbar$. The potential of this force over the range r_p is equal to $U = F_p r_p = m_p c^2$.

Because the dense assembly of positive and negative Planck mass particles defines an absolute system at rest with these particles, one may speak of an aether composed of densely packed Planck mass particles which one may simply call the Planck aether.

A repulsive force acting between two positive Planck mass particles will repel the particles, and an attractive force acting between two negative Planck mass particles will do the same because for a negative mass the direction of the acceleration is opposite to the direction of the force. According to the assumption that a positive Planck mass particle exerts a repulsive and a negative Planck mass particle an attractive force, a positive and negative Planck mass particle should be accelerated towards each other. With the negative mass thereby accelerated in a direction opposite to the direction of the acceleration for the positive mass, the law of linear momentum conservation seems to be violated even though the energy is conserved during the entire collision process, with the sum of kinetic and potential energies remaining unchanged. However, within the infinitely large assembly of positive and negative Planck mass particles, the total momentum can be conserved through recoil imparted onto this assembly even though it may be violated individually for one positive Planck mass particle colliding with one negative Planck mass particle. The force between positive and negative Planck mass particles is for this reason generated through the constraint that each Planck length volume in space shall (in the average) be occupied by one Planck mass particle.* From this perspective, the force between two Planck mass particles of opposite sign should be viewed as the force between a particle and a hole, with a negative Planck mass particle acting like a hole on a positive Planck mass particle, and a positive Planck mass particle acting like a hole on a negative Planck mass particle. As it is known from solid state physics, the interaction of a particle with a hole can violate Newton's actio = reactio postulate, but not for the solid as a whole. The assumed force law is different from the force law if the Planck mass particles would be the source of a Newtonian gravitational field. The proposed theory rather seeks to explain all fields, including the gravitational field, and all elementary particles as quasiparticles resulting from collective excitations of the Planck aether. As in condensed matter physics, where the attractive force of phonon fields has its cause in the repulsive short range force between the molecules, long range forces, like electromagnetic and gravitational forces, are conjectured to have their cause in the short range forces between the Planck mass particles. This possibility to explain all long range forces by collective excitations of a medium having their cause in short range forces, frees one from the need to impose a Newtonian gravitational force law between the Planck mass particles.

A vacuum composed of Planck mass particles subject to Newton's (or Einstein's) law of gravity would be unstable even in the absence of negative masses, because an assembly of positive Planck masses alone would already be unstable against gravitational collapse. The addition of negative masses would increase the instability by self-accelerating pairs of positive and negative masses. Einstein's gravitational field equations require that the quantum mechanical zero point energy of the vacuum must be cut off at the gravitational radius of this energy. This would lead to a vacuum where each Planck length volume is occupied by a Planck mass black hole. As in the case of a vacuum densely occupied by positive Planck mass particles, this not only makes the vacuum unstable, but results

* Imposing such a constraint is reminiscent of the efforts by H. Hertz (Principles of Mechanics, Macmillan, New York 1899) who tried to eliminate the force concept altogether, replacing it by coupling the system under consideration with hidden masses interacting with the observed masses through constraints. In our model, where everything is reduced to Planck units, the force, or what it replaces, must be expressed in Planck units with a range equal to a Planck length. This is a force quite different from Newton's hard core force which cannot be expressed in terms of Planck units.

in a vacuum mass density of $\sim 10^{95}$ g/cm^3, a physical impossibility.

The proposed alternative force law between the Planck mass particles not only makes the assembly of the positive and negative Planck mass particles stable, but gives this assembly a striking similarity to condensed matter, where electric charges of equal sign repel and those of opposite sign attract each other.

The many similarities and analogies of high energy physics with condensed matter physics, where even a Lorentz invariance can be established for phonon fields (by replacing the velocity of light with the velocity of sound) rather suggests that the assembly of Planck masses supposedly filling the vacuum behaves like a condensed matter plasma, where the positive and negative charges are replaced by positive and negative masses, with the masses of equal sign repelling and those of opposite sign attracting each other.

2. Newtonian Mechanics of the Planck Mass Particles

Because two negative Planck mass particles repel each other as do two positive Planck mass particles, the outcome of a collision between two negative Planck mass particles is the same as between two positive Planck mass particles, but the outcome of a collision between a positive and negative Planck mass particle, to be viewed as the collision with a hole, is quite different. A positive Planck mass particle coming into contact with such a hole is pushed into the hole by the pressure of all the other positive Planck mass particles surrounding it. Likewise, a negative Planck mass particle is pushed into a hole formed by a positive Planck mass particle. As a result, the encounter of a positive with a negative Planck mass particle leads to an acceleration of the positive towards the negative Planck mass particle. The force leading to this acceleration has its cause in the localized pressure gradients set up by the presence of a hole. However, because these forces are transmitted with the velocity of sound they are subject to an aberration. In the proposed theory, sound waves are propagated through the Planck aether with the velocity of light, and the aberration depends on the ratio of the particle velocity to the velocity of light.

The aberration vanishes if the particle interacting with a hole is at relative rest to the hole. The collision process can then simply be described the interaction of a particle with a hole of diameter r_p. For a positive Planck mass particle falling into the hole formed by a negative Planck mass particle, the pressure gradient force is given by

$$f = -r_p^3 \nabla p_+ , \tag{2.1}$$

where $p_+ = n_+ m_p c^2 = (1/2\, r_p^3)\, m_p c^2$, with $n_+ = 1/2\, r_p^3$ the number density of positive Planck masses. With $n = n_+ + n_- = 1/r_p^3$, the number density of the negative Planck mass particles is $n_- = 1/2\, r_p^3$. During the encounter with the positive Planck mass particle the negative Planck mass particle is subject to the same pressure gradient force, because for it the pressure is $p_- = -n_- m_p c^2$, with $\nabla p_- = \nabla p_+$. The magnitude of the pressure gradient generated by the void of the negative Planck mass acting on the positive Planck mass, and vice versa, is $|\nabla p_\pm| = |p_\pm|/r_p$, and hence the force acting on both the positive and negative Planck mass particle is

$$f = r_p^2 \, |p_\pm| = (c^2/2)\,(m_p/r_p) = (1/2)\, c^4/G = (1/2)\, F_p . \tag{2.2}$$

In a frame at rest with one particle, the force on the other particle can then be viewed as equal to F_p.

The interaction of a positive (or negative) Planck mass particle with a hole formed by a negative (or positive) Planck mass particle can most easily be analyzed as a collision with a potential well of diameter r_p and depth $U = r_p F_p = m_p c^2$. One has to consider then only two cases: First, a central, and second, a glancing collision. Any other case can be seen as a combination of these two limiting cases.

In the first case, the acceleration is given by $a_p = c/t_p = c^2/r_p = F_p/m_p$ and one obtains for the displacement of the particle trajectory

$$\begin{aligned} \delta &= \pm(1/2)\, a_p t_p^2 = \pm(1/2)\, r_p , \\ \dot{\delta} &= \pm a_p t_p = \pm c . \end{aligned} \tag{2.3 a}$$

It is equal to the "Zitterbewegung" displacement derived by Schrödinger for the Dirac electron, where it is caused by the negative energy, and hence negative mass states of the Dirac equation. The worldlines for this displacement are shown in Fig. 1a, both for a positive and negative Planck mass particle.

For the second case, it is convenient to place the hole in a position at rest with regard to the incoming particle tangentially striking the hole with the velocity $v_\perp$, perpendicular to the direction from the center of the hole to the point of impact. The angle α by which the pressure gradient force suffers an aberration from

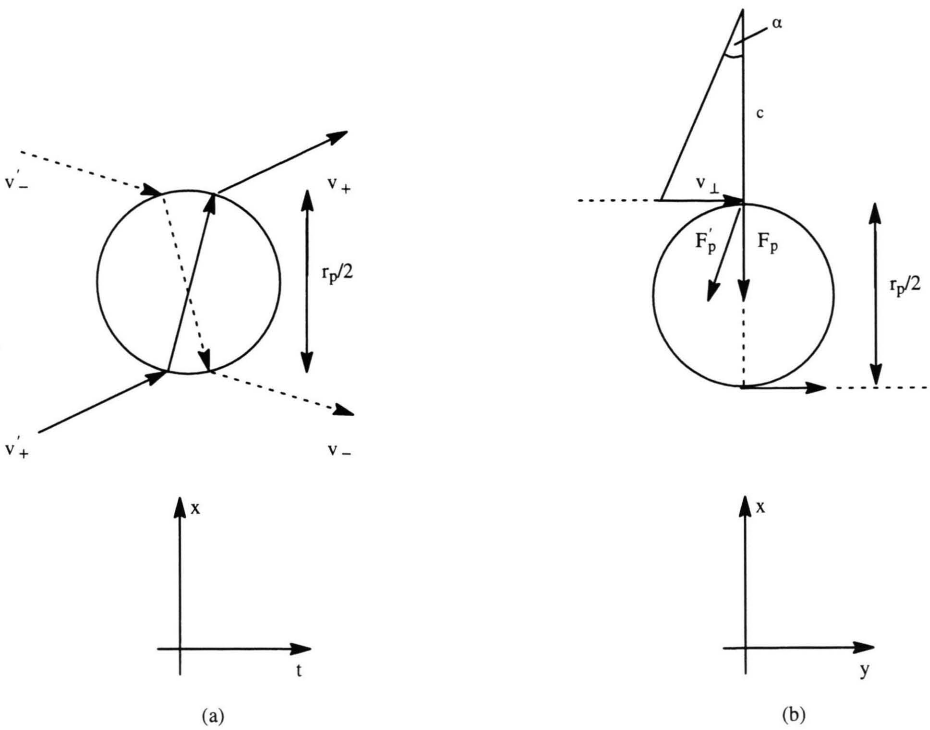


Fig. 1. "Zitterbewegung" displacements resulting from the collision of a positive with a negative Planck mass particle. The negative particle acts with regard to the positive particle like a hole, and vice versa.

its direction towards the center of the hole is given by $\tan\alpha = v_\perp/c$, whereby F_p is changed into $F_p' = F_p/\cos\alpha$ (see Figure 1 b). Accordingly, the force component directed towards the center of the hole is still equal to F_p, but it is now accompanied by a force component directed perpendicular to F_p and opposite to $v_\perp$, having the magnitude $F_{p\perp} = F_p \tan\alpha = (v_\perp/c)/F_p$. This force leads to a displacement perpendicular to the direction of $v_\perp$:

$$\delta_\perp = (1/2)\,(F_{p\perp}/m_p)\, t_p^2 = (v_\perp/c)\,(1/2)\, r_p\,, \qquad \dot{\delta}_\perp = v_\perp\,. \tag{2.3b}$$

At the end of the collision process, when the particle leaves the potential well, an equal but opposite force to $F_{p\perp}$ restores the particle velocity to $v_\perp$, nullifying the advance $v_\perp t_p$ the particle would otherwise make into the y-direction (see Figure 1 b).

With the kinetic energy and linear momentum returning to their initial values after completion of the collision between a positive and negative Planck mass particle, the conservation laws for energy and momentum imply that

$$\boldsymbol{v}_+^2 - \boldsymbol{v}_-^2 = \boldsymbol{v}_+'^2 - \boldsymbol{v}_-'^2\,, \qquad \boldsymbol{v}_+ - \boldsymbol{v}_- = \boldsymbol{v}'_+ - \boldsymbol{v}'_-\,, \tag{2.4}$$

where $\boldsymbol{v}'_\pm$ are the velocities before and $\boldsymbol{v}_\pm$ those after the encounter. From (2.4) it follows that

$$\boldsymbol{v}_\pm = \boldsymbol{v}'_\pm\,, \tag{2.5}$$

as if the particles had gone through each other with no effect at all. For the collision of a positive and equal in magnitude negative mass the situation though is different because the center of mass of such a mass dipole is located at ∞. The center of mass of two colliding particles has six integrals, three for its velocity and three for its position at a given instant in time. Since for a mass dipole the center of mass is located at ∞, the colliding particles may for this reason suffer a parallel displacement of their trajectories.

During the collision of a positive with a negative Planck mass particle, the momentum of each Planck mass particle fluctuates by $\Delta p = m_p c$, with the total fluctuation in momentum $2\,m_p c$ compensated by the recoil to the positive and negative Planck mass fluid. This momentum fluctuation is accompanied by an energy fluctuation $\Delta E = m_p c^2$, hence

$$\Delta p = \hbar/r_p, \quad \Delta E = \hbar/t_p. \tag{2.6}$$

Heisenberg's uncertainty relations for momentum and energy are thus explained by the mechanical fluctuations of the positive-negative Planck mass particle fluid, and it is for this reason of no surprise that Schrödinger's equation for a Planck mass particle can be derived from the Boltzmann equation for such a fluid.

The Boltzmann equation is given by [1]

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \boldsymbol{a} \cdot \frac{\partial f}{\partial \boldsymbol{v}} = \int v_{rel} (f' f_1' - f f_1)\, d\sigma\, d\boldsymbol{v}_1, \tag{2.7}$$

where f is the distribution function of the colliding particles, f', f_1' before and f, f_1 after the collision, with f_1' and f_1 the distribution functions of the particles which by colliding with those particles belonging to f' and f change the distribution function from f' to f. The magnitude of the relative collision velocity is v_{rel}, and the collision cross section is σ. The particle number density is $n(\boldsymbol{r}, t) = \int f(\boldsymbol{v}, \boldsymbol{r}, t)\, d\boldsymbol{v}$ and the average velocity $\boldsymbol{V} = \int \boldsymbol{v} f(\boldsymbol{v}, \boldsymbol{r}, t)\, d\boldsymbol{v}/n(\boldsymbol{r}, t)$. The acceleration is $\boldsymbol{a} = \mp(1/m_p)\nabla U$, where $U(\boldsymbol{r})$ is the potential of a force.

The Boltzmann equation for the distribution function $f_\pm$ of the positive and negative Planck mass particles is

$$\frac{\partial f_\pm}{\partial t} + \boldsymbol{v}_\pm \cdot \frac{\partial f_\pm}{\partial \boldsymbol{r}} \mp \frac{1}{m_p} \frac{\partial U}{\partial \boldsymbol{r}} \cdot \frac{\partial f_\pm}{\partial \boldsymbol{v}_\pm} = 4\alpha c\, r_p^2 \int (f_\pm' f_\mp' - f_\pm f_\mp)\, d\boldsymbol{v}_\mp, \tag{2.8}$$

where we have set $\sigma = (2r_p)^2 = 4r_p^2$ and $v_{rel} = \alpha c$, with α a numerical factor. In (2.8) U describes here the average potential of all Planck mass particles on one Planck mass. The constraint of keeping constant the average number density of all Planck mass particles leads to a pressure which has to be included in the potential U. It can be viewed as a potential holding together the positive and negative Planck mass particles, which otherwise would fly apart. The effective interaction between the positive and negative Planck mass particles caused by the constraint in the Boltzmann equation is separated into the short range "Zitterbewegung" part entering the collision integral and the long range average potential part included in the potential U.

Because of (2.5) one has

$$f_\pm'(\boldsymbol{r}) = f_\pm(\boldsymbol{r} \pm \boldsymbol{r}_p/2), \tag{2.9}$$

where one has to average over all possible displacements and velocities of the "Zitterbewegung". Because the distribution function f' before the collision is set equal the displaced distribution function f, the direction of the "Zitterbewegung" velocity is in the opposite direction of the displacement vector $\boldsymbol{r}_p/2$. With (2.9) the integrand in the collision integral becomes

$$f_\pm' f_\mp' - f_\pm f_\mp = f_\pm\left(\boldsymbol{r} \pm \frac{\boldsymbol{r}_p}{2}\right) f_\mp\left(\boldsymbol{r} \mp \frac{\boldsymbol{r}_p}{2}\right) - f_\pm(\boldsymbol{r}) f_\mp(\boldsymbol{r}). \tag{2.10}$$

Expanding $f_\pm\left(\boldsymbol{r} \pm \frac{\boldsymbol{r}_p}{2}\right)$ and $f_\mp\left(\boldsymbol{r} \mp \frac{\boldsymbol{r}_p}{2}\right)$ into a Taylor series,

$$f_\pm\left(\boldsymbol{r} \pm \frac{\boldsymbol{r}_p}{2}\right) = f_\pm \pm \frac{\boldsymbol{r}_p}{2} \cdot \frac{\partial f_\pm}{\partial \boldsymbol{r}} + \frac{r_p^2}{8} \frac{\partial^2 f_\pm}{\partial \boldsymbol{r}^2} + \ldots,$$
$$f_\mp\left(\boldsymbol{r} \mp \frac{\boldsymbol{r}_p}{2}\right) = f_\mp \mp \frac{\boldsymbol{r}_p}{2} \cdot \frac{\partial f_\mp}{\partial \boldsymbol{r}} + \frac{r_p^2}{8} \frac{\partial^2 f_\mp}{\partial \boldsymbol{r}^2} + \ldots, \tag{2.11}$$

one finds up to second order that

$$f_\pm' f_\mp' - f_\pm f_\mp \simeq \pm \frac{\boldsymbol{r}_p}{2} \cdot \left(f_\mp \frac{\partial f_\pm}{\partial \boldsymbol{r}} - f \pm \frac{\partial f_\mp}{\partial \boldsymbol{r}} \right) - \frac{r_p^2}{4} \frac{\partial f_\pm}{\partial \boldsymbol{r}} \cdot \frac{\partial f_\mp}{\partial \boldsymbol{r}} + \frac{r_p^2}{8} \cdot \left(f_\mp \frac{\partial^2 f_\pm}{\partial \boldsymbol{r}^2} + f_\pm \frac{\partial^2 f_\mp}{\partial \boldsymbol{r}^2} \right) \tag{2.12}$$

with higher order terms suppressed by the Planck length. Because approximately $f_\mp(\boldsymbol{v}_\mp, \boldsymbol{r}, t) \simeq f_\pm(\boldsymbol{v}_\pm, \boldsymbol{r}, t)$, one has

$$f_\pm' f_\mp' - f_\pm f_\mp \simeq \frac{r_p^2}{4} \left(\frac{\partial f_\pm}{\partial \boldsymbol{r}} \right)^2 + \frac{r_p^2}{4} f_\pm \frac{\partial^2 f_\pm}{\partial \boldsymbol{r}^2} = \left(\frac{r_p}{2} \right)^2 f_\pm^2 \frac{\partial^2 \log f_\pm}{\partial \boldsymbol{r}^2} \simeq \left(\frac{r_p}{2} \right)^2 f_\pm f_\mp \frac{\partial^2 \log f_\pm}{\partial \boldsymbol{r}^2}. \tag{2.13}$$

To obtain the net "Zitterbewegung" displacement over a sphere with a volume to surface ratio $(r_p/2)^3/(r_p/2)^2 = r_p/2$, (2.13) must be multiplied by the operator $(1/2)\,\boldsymbol{r}_p \cdot \partial/\partial \boldsymbol{r}$, and to obtain the corresponding net value in velocity space it must in addition be multiplied by the operator $\boldsymbol{c} \cdot \partial/\partial \boldsymbol{v}_\pm$, with the vector $\boldsymbol{c}$ in opposite direction to $\boldsymbol{r}_p$.

Integrating the r.h.s of (2.8) over $dv_\mp$, and setting $\int f_\mp \, d\boldsymbol{v}_\mp \simeq 1/2 \, r_p^3$, the number density of one Planck mass species in the undisturbed configuration of the Planck mass particles filling space, one then has

$$\frac{\partial f_\pm}{\partial t} + \boldsymbol{v}_\pm \cdot \frac{\partial f_\pm}{\partial \boldsymbol{r}} \mp \frac{1}{m_p} \frac{\partial U}{\partial \boldsymbol{r}} \cdot \frac{\partial f_\pm}{\partial \boldsymbol{v}_\pm} = -\frac{\alpha c^2 r_p^2}{4} \frac{\partial^2}{\partial \boldsymbol{v}_\pm \, \partial \boldsymbol{r}} \left(f_\pm \frac{\partial^2 \log f_\pm}{\partial \boldsymbol{r}^2} \right). \tag{2.14}$$

For an approximate solution of (2.14) one computes its zeroth and first moment. The zeroth moment is obtained by integrating (2.14) over $d\boldsymbol{v}_\pm$, with the result that

$$\frac{\partial n_\pm}{\partial t} + \frac{\partial (n_\pm \boldsymbol{V}_\pm)}{\partial \boldsymbol{r}} = 0, \tag{2.15}$$

which is the continuity equation for the macroscopic quantities $n_\pm$ and $\boldsymbol{V}_\pm$. The first moment is obtained by multiplying (2.14) with $\boldsymbol{v}_\pm$ and integrating over $d\boldsymbol{v}_\pm$. Because the logarithmic dependence can be written with sufficient accuracy as $\partial^2 \log f_\pm / \partial \boldsymbol{r}^2 \simeq \partial^2 \log n_\pm / \partial \boldsymbol{r}^2$, one finds

$$\frac{\partial (n_\pm \boldsymbol{V}_\pm)}{\partial t} + \frac{\partial (n_\pm \boldsymbol{V}_\pm \cdot \boldsymbol{V}_\pm)}{\partial \boldsymbol{r}} = \mp \frac{n_\pm}{m_p} \frac{\partial U}{\partial \boldsymbol{r}} + \frac{\alpha c^2 r_p^2}{4} \frac{\partial}{\partial \boldsymbol{r}} \left(n_\pm \frac{\partial^2 \log n_\pm}{\partial \boldsymbol{r}^2} \right). \tag{2.16}$$

With the help of (2.15) this can be written as

$$\frac{\partial \boldsymbol{V}_\pm}{\partial t} + \boldsymbol{V}_\pm \cdot \frac{\partial \boldsymbol{V}_\pm}{\partial \boldsymbol{r}} = \mp \frac{1}{m_p} \frac{\partial U}{\partial \boldsymbol{r}} + \frac{\alpha \hbar^2}{4 m_p^2 n_\pm} \frac{\partial}{\partial \boldsymbol{r}} \left(n_\pm \frac{\partial^2 \log n_\pm}{\partial \boldsymbol{r}^2} \right), \tag{2.17}$$

for which one can also write

$$\frac{\partial \boldsymbol{V}_\pm}{\partial t} + \boldsymbol{V}_\pm \cdot \frac{\partial \boldsymbol{V}_\pm}{\partial \boldsymbol{r}} = \mp \frac{1}{m_p} \frac{\partial U}{\partial \boldsymbol{r}} + \frac{\alpha \hbar^2}{2 m_p^2} \frac{\partial}{\partial \boldsymbol{r}} \left(\frac{1}{\sqrt{n_\pm}} \frac{\partial^2 \sqrt{n_\pm}}{\partial \boldsymbol{r}^2} \right). \tag{2.18}$$

The equivalence of (2.15) and (2.18) with the one-body Schrödinger equation for a positive or negative Planck mass can now be established by Madelung's transformation

$$n_\pm = \psi_\pm^* \psi_\pm,$$
$$n_\pm \boldsymbol{V}_\pm = \mp \frac{i\hbar}{2 m_p} [\psi_\pm^* \nabla \psi_\pm - \psi_\pm \nabla \psi_\pm^*], \tag{2.19}$$

transforming Schrödinger's equation of a Planck mass $\pm m_p$

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2 m_p} \nabla^2 \psi_\pm + U(\boldsymbol{r}) \psi_\pm \tag{2.20}$$

into

$$\frac{\partial n_\pm}{\partial t} + \frac{\partial (n_\pm \boldsymbol{V}_\pm)}{\partial \boldsymbol{r}} = 0,$$
$$\frac{\partial \boldsymbol{V}_\pm}{\partial t} + \boldsymbol{V}_\pm \cdot \frac{\partial \boldsymbol{V}_\pm}{\partial \boldsymbol{r}} = \mp \frac{1}{m_p} \frac{\partial}{\partial \boldsymbol{r}} [U + Q_\pm], \tag{2.21}$$

where

$$Q_\pm = \mp \frac{\hbar^2}{2 m_p} \frac{1}{\sqrt{n_\pm}} \frac{\partial^2 \sqrt{n_\pm}}{\partial \boldsymbol{r}^2} \tag{2.22}$$

is the so called quantum potential. By comparison with (2.15) and (2.18) one finds full equivalence for $\alpha = 1$, that is for $v_{rel} = c$.

The importance of this result is that quantum mechanics is shown to have its cause in the existence of negative masses at the Planck scale. Another important result is that quantum mechanics becomes invalid for masses large compared to m_p and must be replaced by Newtonian mechanics, since the negative Planck masses of the vacuum cannot exert an appreciable "Zitterbewegung" on a mass $m \gg m_p$. The uncertainty in quantum mechanics is here not seen due to a fundamental noncausal structure, but rather the consequence of the principal inability to make measurements for distances and times smaller than r_p and t_p, not permitting to calculate the otherwise deterministic outcome of the collisions between Planck mass particles which would require the knowledge obtained from such measurements.

Taking in the collision integral into account higher order terms otherwise suppressed by the Planck length, we have (by assuming that $f_\pm \simeq f_\mp$)

$$\begin{aligned}
\log f'_\pm f'_\mp &= \log f_\pm \left(\boldsymbol{r} \pm \frac{\boldsymbol{r}_p}{2} \right) + \log f_\mp \left(\boldsymbol{r} \mp \frac{\boldsymbol{r}_p}{2} \right) \\
&\simeq \log f_\pm \left(\boldsymbol{r} \pm \frac{\boldsymbol{r}_p}{2} \right) + \log f_\pm \left(\boldsymbol{r} \mp \frac{\boldsymbol{r}_p}{2} \right) \\
&= \exp\left(\pm \frac{\boldsymbol{r}_p}{2} \cdot \frac{\partial}{\partial \boldsymbol{r}} \right) [\log f_\pm] + \exp\left(\mp \frac{\boldsymbol{r}_p}{2} \cdot \frac{\partial}{\partial \boldsymbol{r}} \right) [\log f_\pm] \\
&= 2\cosh\left(\pm \frac{\boldsymbol{r}_p}{2} \cdot \frac{\partial}{\partial \boldsymbol{r}} \right) [\log f_\pm] \\
&= \cosh\left(\mp \frac{\boldsymbol{r}_p}{2} \cdot \frac{\partial}{\partial \boldsymbol{r}} \right) [\log f_\pm^2],
\end{aligned} \tag{2.23}$$

where $\frac{r_p}{2}\cdot\frac{\partial}{\partial \boldsymbol{r}}$ is an operator for which $\left(\frac{r_p}{2}\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)^n=\left(\frac{r_p}{2}\right)^n\cdot\frac{\partial^n}{\partial \boldsymbol{r}^n}$. We thus have

$$f'_{\pm}\,f'_{\mp}-f_{\pm}\,f_{\mp} \simeq \exp\left\{\cosh\left(\pm\frac{r_p}{2}\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)[\log f_{\pm}^2]-f_{\pm}^2. \tag{2.24}$$

To obtain from (2.23) the approximate expression (2.13), we expand the hyperbolic function up to the second order and the exponential function up to first order:

$$\exp\{\,\}-f_{\pm}^2=\exp\left\{\log f_{\pm}^2+\left(\frac{r_p}{2}\right)^2\frac{\partial^2\log f_{\pm}}{\partial \boldsymbol{r}^2}\right\}-f_{\pm}^2 \simeq\left(\frac{r_p}{2}\right)^2 f_{\pm}^2\,\frac{\partial^2\log f_{\pm}}{\partial \boldsymbol{r}^2}, \tag{2.25}$$

which is the same as (2.13).

Inserting (2.24) into (2.8), putting $\alpha=1$, applying the operator $(r_p/2)\,c\,\partial^2/\partial \boldsymbol{v}_{\pm}\,\partial \boldsymbol{r}$, finally integrating over $d\boldsymbol{v}_{\mp}$, whereby $\int f_{\mp}\,d\boldsymbol{v}_{\mp}\simeq 1/2\,r_p^3$, one has

$$\frac{\partial f_{\pm}}{\partial t}+\boldsymbol{v}_{\pm}\cdot\frac{\partial f_{\pm}}{\partial \boldsymbol{r}}\mp\frac{1}{m_p}\frac{\partial U}{\partial \boldsymbol{r}}\cdot\frac{\partial f_{\pm}}{\partial \boldsymbol{v}_{\pm}}=-c^2\frac{\partial^2}{\partial \boldsymbol{v}_{\pm}\,\partial \boldsymbol{r}}\cdot\left\{f_{\pm}\left[\frac{\exp\left\{\cosh\left(\pm\frac{r_p}{2}\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)[\log f_{\pm}^2]\right\}}{f_{\pm}^2}-1\right]\right\}. \tag{2.26}$$

Integrating (2.26) over $d\boldsymbol{v}_{\pm}$, one obtains the continuity equation (2.15). Multiplying (2.26) by $\boldsymbol{v}_{\pm}$, integrating over $d\boldsymbol{v}_{\pm}$, and setting $\partial^2\log f_{\pm}^2/\partial \boldsymbol{r}^2\simeq\partial^2\log n_{\pm}^2/\partial \boldsymbol{r}^2$, one obtains

$$\frac{\partial(n_{\pm}\boldsymbol{V}_{\pm})}{\partial t}+\frac{\partial(n_{\pm}\boldsymbol{V}_{\pm}\cdot\boldsymbol{V}_{\pm})}{\partial \boldsymbol{r}}=\mp\frac{n_{\pm}}{m_p}\frac{\partial U}{\partial \boldsymbol{r}}+c^2\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{n_{\pm}\left[\frac{\exp\left\{\cosh\left(\pm\frac{r_p}{2}\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)[\log n_{\pm}^2]\right\}}{n_{\pm}^2}-1\right]\right\}, \tag{2.27}$$

which by (2.15) can be simplified:

$$\frac{\partial \boldsymbol{V}_{\pm}}{\partial t}+\boldsymbol{V}_{\pm}\cdot\frac{\partial \boldsymbol{V}_{\pm}}{\partial \boldsymbol{r}}=\mp\frac{1}{m_p}\frac{\partial U}{\partial \boldsymbol{r}}+\frac{c^2}{n_{\pm}}\frac{\partial}{\partial \boldsymbol{r}}\cdot\left\{n_{\pm}\left[\frac{\exp\left\{\cosh\left(\pm\frac{r_p}{2}\cdot\frac{\partial}{\partial \boldsymbol{r}}\right)[\log n_{\pm}^2]\right\}}{n_{\pm}^2}-1\right]\right\}. \tag{2.28}$$

By applying the inverted Madelung transformation on (2.28) one can obtain higher order correction terms for the Schrödinger equation, including nonlinear terms suppressed by the Planck length.

3. Quantum Mechanics of the Densely Packed Assembly of Positive and Negative Planck Mass Particles

Having established quantum mechanics for a single Planck mass particle within a dense assembly of positive and negative Planck mass particles, a quantum mechanical description of the many body problem for all the Planck mass particles can be given. It is achieved 1) by setting the potential U in (2.20) equal to

$$U=2\hbar\,c\,r_p^2[\psi_{+}^{*}\,\psi_{+}-\psi_{-}^{*}\,\psi_{-}], \tag{3.1}$$

2) by replacing the field functions $\psi_{\pm}$, $\psi_{\pm}^{*}$ by the operators $\psi_{\pm}$, $\psi_{\pm}^{\dagger}$ obeying the canonical commutation relations

$$[\psi_{\pm}(\boldsymbol{r})\,\psi_{\pm}^{\dagger}(\boldsymbol{r}')]=\delta(\boldsymbol{r}-\boldsymbol{r}'),$$
$$[\psi_{\pm}(\boldsymbol{r})\,\psi_{\pm}(\boldsymbol{r}')]=[\psi_{\pm}^{\dagger}(\boldsymbol{r})\,\psi_{\pm}^{\dagger}(\boldsymbol{r}')]=0, \tag{3.2}$$

whereby (2.20) becomes the operator field equation

$$i\hbar\frac{\partial\psi_{\pm}}{\partial t}=\mp\frac{\hbar^2}{2m_p}\nabla^2\psi_{\pm}\pm 2\hbar\,c\,r_p^2(\psi_{\pm}^{\dagger}\,\psi_{\pm}-\psi_{\mp}^{\dagger}\,\psi_{\mp})\,\psi_{\pm}. \tag{3.3}$$

We justify it as follows: 1) An undisturbed dense assembly of Planck mass particles, each occupying the volume r_p^3, has the expectation value $\langle\psi_{\pm}^{\dagger}\,\psi_{\pm}\rangle=1/2r_p^3$ whereby $2\hbar c\,r_p^2\langle\psi_{\pm}^{\dagger}\,\psi_{\pm}\rangle=m_p\,c^2$, implying an average potential energy $\pm m_p\,c^2$ for the positive and negative Planck mass particles within the assembly of all Planck masses. This is consistent with the value of the potential $F_p\,r_p=m_p\,c^2$ of the Planck force acting over the distance r_p. The interaction term between the positive and negative Planck mass fluid results from the constraint demanding that the number density of Planck masses shall (in the average) be equal to $1/r_p^3$. 2) The rules of quantum mechanics for one Planck mass imply the one-particle commutation rule $[p,q]=\hbar/i$, which for a many-particle system of Planck mass particles leads to the canonical commutation relation (3.2) applied to the operator field equation (3.3) describing the many Planck mass system.

Equation (3.3) was the proposed Planck aether model for a theory of elementary particles, with some of the following results previously published in several papers in this journal (1990–1995). To solve (3.3) one has to employ suitable approximation methods. In its groundstate, the system of Planck mass particles can be viewed as a two-component positive-negative-mass superfluid, in which each component is described by a completely symmetric wave function. Because of the exchange integral, the absolute value of the interaction energy between identical locally (point-like) interacting Planck mass particles is there twice as large. In the Hartree-Fock approximation where this exchange interaction is taken into account, one has for this reason to replace the product of three field operators for equal particles by twice their expectation value, whereby the operator field equation (3.3) leads to the nonlinear Schrödinger equation

$$i\hbar\frac{\partial\psi_\pm}{\partial t}=\mp\frac{\hbar^2}{2m_p}\nabla^2\psi_\pm \pm 2\hbar c r_p^2[2\psi_\pm^*\psi_\pm-\psi_\mp^*\psi_\mp]\psi_\pm . \tag{3.4}$$

By the Madelung transformation (2.19) one obtains for (3.4)

$$\frac{\partial \mathbf{V}_\pm}{\partial t}+(\mathbf{V}_\pm\cdot\nabla)\mathbf{V}_\pm=-2c^2r_p^3\nabla(2n_\pm-n_\mp)\mp\frac{1}{m_p}\nabla Q_\pm$$

$$\frac{\partial n_\pm}{\partial t}+\nabla\cdot(n_\pm\mathbf{V}_\pm)=0, \tag{3.5}$$

where

$$\psi_\pm=A_\pm e^{iS_\pm},\ A_\pm>0,\quad 0<S_\pm<2\pi,$$

$$n_\pm=A_\pm^2,\quad \mathbf{V}_\pm=\pm\frac{\hbar}{m_p}\nabla S_\pm \tag{3.6}$$

with $Q_\pm$ as in (2.22). It thus follows that

$$\oint \mathbf{V}_\pm\cdot d\mathbf{r}=\pm n\,h/m_p,\quad n=0,1,2,\ldots. \tag{3.7}$$

Small amplitude time-dependent solutions with a wave length large compared to the DeBroglie wave length of the Planck mass particles are obtained by linearizing (3.5) and neglecting the quantum potential:

$$\frac{\partial}{\partial t}(\mathbf{V}_+ + \mathbf{V}_-)=-2c^2r_p^3\nabla(n'_+ + n'_-),$$

$$\frac{\partial}{\partial t}(\mathbf{V}_+ - \mathbf{V}_-)=-6c^2r_p^3\nabla(n'_+ - n'_-),$$

$$\frac{\partial n'_\pm}{\partial t}+n_\pm\nabla\cdot\mathbf{V}_\pm=0, \tag{3.8}$$

where $n'_\pm$ is small disturbance imposed on the undisturbed value $n_\pm=1/2r_p^3$. Eliminating $n'_\pm$ one has two equations for longitudinal waves:

$$\frac{\partial^2}{\partial t^2}(\mathbf{V}_+ + \mathbf{V}_-)=c^2\nabla^2(\mathbf{V}_+ + \mathbf{V}_-),$$

$$\frac{\partial^2}{\partial t^2}(\mathbf{V}_+ - \mathbf{V}_-)=-3c^2\nabla^2(\mathbf{V}_+ - \mathbf{V}_-). \tag{3.9}$$

The first describes waves propagating with the velocity of light. It has the property of an acoustic type wave. The second wave propagating with $\sqrt{3}c$, conserves the total number density of the Planck mass particles with the positive Planck mass particles oscillating against the negative Planck mass particles. It resembles electrostatic plasma oscillations.

In the limit of short wave lengths one has

$$\frac{\partial \mathbf{V}_\pm}{\partial t}=-\frac{1}{m_p}\nabla Q_\pm, \tag{3.10}$$

whereby (2.21) and (3.5) leads to

$$\frac{\partial^2\mathbf{V}_\pm}{\partial t^2}=-\frac{\hbar^2}{4m_p^2}\nabla^4\mathbf{V}_\pm, \tag{3.11}$$

which has the dispersion relation $\omega=\pm\hbar k^2/2m_p$ of the Schrödinger equation for a single Planck mass.

Stationary solutions are obtained from (3.5) putting $\partial/\partial t=0$. With the boundary condition $\mathbf{V}_\pm=0$ for $r\to\infty$, and neglecting $Q_\pm$ one obtains

$$\frac{\mathbf{V}_+^2+\mathbf{V}_-^2}{2}=-2c^2[r_p^3(n_+ + n_-)-1],$$

$$\frac{\mathbf{V}_+^2-\mathbf{V}_-^2}{2}=-6c^2r_p^3(n_+ - n_-). \tag{3.12}$$

Because curl $\mathbf{V}_\pm=0$, (3.7) for $n=1$ in cylindrical coordinates leads to the line vortex solution:

$$|\mathbf{V}_\pm|=c(r_p/r),\quad r>r_p,$$
$$=0\quad ,\quad r<r_p. \tag{3.13}$$

The cut-off $|\mathbf{V}_\pm|=0$ for $r<r_p$ results from the quantum potential at $r=r_p$, $|Q_\pm(r_p)|/m_p\simeq\hbar^2/(2m_p^2r_p^2)=c^2/2$, which by setting it equal to $\mathbf{V}_\pm^2/2$ leads to $\mathbf{V}_\pm=c$ at $r=r_p$.

Inserting (3.13) into (3.12) one has

$$\mathbf{V}_-=\pm\mathbf{V}_+,\quad n_+ + n_-=0,$$
$$n_\pm=(1/2r_p^3)[1-(1/2)(r_p/r)^2], \tag{3.14}$$

describing two double vortex configurations: One where $\mathbf{V}_-=\mathbf{V}_+$, with the positive and negative masses corotating, and one where $\mathbf{V}_-=-\mathbf{V}_+$, where they are counterrotating.

For ring vortices with a ring radius $R \gg r_p$, the vortices have the resonance frequency

$$\omega_v \simeq c\, r_p / R^2 \tag{3.15}$$

In the limit $R \to r_p$, the ring vortices degenerate into rotons.

Because space is filled with an equal number of positive and negative Planck mass particles, a large number of vortex pairs can be created without the expenditure of energy. A configuration of this kind, called a vortex sponge, permits the propagation of two types of transverse waves: One simulating Maxwell's electromagnetic waves (first recognized by Thomson in 1887), and the other one Einstein's gravitational waves.

To show this, let $\boldsymbol{V} = \{V_x, V_y, V_z\}$ be the undisturbed velocity of the vortex field and $\boldsymbol{u} = \{u_x, u_y, u_z\}$ a small superimposed velocity disturbance, and let us take only those solutions for which div $\boldsymbol{V} = \mathrm{div}\, \boldsymbol{u} = 0$. The x-component of the equation of motion for a disturbance $\boldsymbol{u}$ is

$$\frac{\partial V_x}{\partial t} + \frac{\partial u_x}{\partial t} = -(V_x + u_x)\frac{\partial (V_x + u_x)}{\partial x} - (V_y + u_y)\frac{\partial (V_x + u_x)}{\partial y} - (V_z + u_z)\frac{\partial (V_x + u_x)}{\partial z} - \frac{1}{\varrho}\frac{\partial p}{\partial x}, \tag{3.16}$$

where p and ϱ are the pressure and density of the fluid making up the vortex sponge. From the continuity equation div $\boldsymbol{V} = 0$, one has

$$V_x \frac{\partial V_x}{\partial x} + V_x \frac{\partial V_y}{\partial y} + V_x \frac{\partial V_z}{\partial z} = 0. \tag{3.17}$$

Subtracting (3.17) from (3.16) and taking the y–z average, one finds

$$\frac{\partial u_x}{\partial t} = -\frac{\partial (\overline{V_y V_x})}{\partial y} - \frac{\partial (\overline{V_z V_x})}{\partial z}, \tag{3.18}$$

and similar expressions for u_y and u_z by taking the x–z and x–y average.

Taking the x-component of the equation of motion, multiplying it by v_y and then taking the y–z average, and the y-component multiplied by v_x taking the x–z average, finally subtracting the first from the second equation one finds

$$\frac{\partial}{\partial t}(\overline{V_x V_y}) = -V^2 \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right), \tag{3.19}$$

where $V^2 = \overline{V_x^2} = \overline{V_y^2} = \overline{V_z^2}$ is the average microvelocity of the vortex field. Putting $\phi_z = -\overline{V_x V_y}/2V^2$, (3.19) is the z-component of

$$\frac{\partial \boldsymbol{\phi}}{\partial t} = \frac{1}{2}\,\mathrm{curl}\, \boldsymbol{u}, \tag{3.20}$$

where $\phi_x = -\overline{V_y V_z}/2V^2$, $\phi_y = -\overline{V_z V_x}/2V^2$. Likewise, (3.18) is the x-component of

$$\frac{\partial \boldsymbol{u}}{\partial t} = -2V^2\,\mathrm{curl}\,\boldsymbol{\phi}. \tag{3.21}$$

Elimination of $\boldsymbol{\phi}$ from (3.20) and (3.21) results in a wave equation for $\boldsymbol{u}$:

$$-(1/V^2)\,\partial^2 \boldsymbol{u}/\partial t^2 + \nabla^2 \boldsymbol{u} = 0. \tag{3.22}$$

In the continuum limit, one has for the microvelocity $V^2 = c^2$. In this limit (3.22) describes a transverse wave propagating with the velocity of light c.

Putting $\boldsymbol{u} = \boldsymbol{E}$ and $\boldsymbol{\phi} = -(1/2c)\,\boldsymbol{H}$, (3.20) and (3.21) have the same form as Maxwell's vacuum field equations

$$-\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} = \mathrm{curl}\,\boldsymbol{E},$$

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \mathrm{curl}\,\boldsymbol{H}. \tag{3.23}$$

To derive the second transverse wave mode, we add (3.17) to (3.16) and take the average over x, y and z:

$$\frac{\partial u_x}{\partial t} = -\frac{\partial \overline{V_x^2}}{\partial x} - \frac{\partial \overline{V_x V_y}}{\partial y} - \frac{\partial \overline{V_x V_z}}{\partial z} \tag{3.24}$$

with similar expressions of u_y and u_z. One can therefore write

$$\frac{\partial u_k}{\partial t} = -\frac{\partial}{\partial x_i}(\overline{V_i V_k}). \tag{3.25}$$

Multiplying the V_i component of the equation of motion with V_k, and vice versa, its V_k-component with V_i, adding both and taking the average, one finds

$$\frac{\partial}{\partial t}(\overline{V_i V_k}) = -V^2\left(\frac{\partial u_i}{\partial x_k} - \frac{\partial u_k}{\partial x_i}\right). \tag{3.26}$$

From (3.25) one has

$$\frac{\partial^2 u_k}{\partial t^2} = -\frac{\partial}{\partial t\, \partial x_i}(\overline{V_i V_k}), \tag{3.27}$$

and from (3.26)

$$\frac{\partial}{\partial x_i \partial t}(\overline{V_i V_k}) = -V^2\left(\frac{\partial}{\partial x_k}\frac{\partial u_i}{\partial x_i} + \frac{\partial^2 u_k}{\partial x_i^2}\right) = -V^2\frac{\partial^2 u_k}{\partial x_i^2}, \tag{3.28}$$

the latter because $\operatorname{div} \boldsymbol{u} = 0$. Eliminating $\overline{V_i V_k}$ from (3.27) and (3.28), and putting as before $V^2 = c^2$, results in

$$\frac{\partial^2 u_k}{\partial t^2} = c^2 \frac{\partial^2 u_k}{\partial x_i^2} \tag{3.29}$$

or

$$\nabla^2 \boldsymbol{u} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = 0. \tag{3.30}$$

To show that (3.30) can describe a gravitational wave propagating in the x-direction, one has to compare the line element of a linearized gravitational wave with the line element describing the deformation of an elastic body. The line element of a gravitational wave is [2]

$$ds^2 = ds_0^2 + h_{22}\,dx_2^2 + 2h_{23}\,dx_2\,dx_3 + h_{33}\,dx_3^2, \tag{3.31}$$

where

$$h_{22} = -h_{33} = f(t - x/c), \quad h_{23} = g(t - x/c) \tag{3.32}$$

with f and g two arbitrary functions, and ds_0^2 the line element in the absence of a gravitational wave. By comparison, the line element describing a deformed body is [3]

$$ds^2 = ds_0^2 + 2\,\varepsilon_{ik}\,dx_i\,ds_k, \tag{3.33}$$

where

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial \varepsilon_i}{\partial x_k} + \frac{\partial \varepsilon_k}{\partial x_i}\right). \tag{3.34}$$

In (3.33) and (3.34) $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_y, \varepsilon_x)$ is the displacement vector related to the velocity disturbance vector $\boldsymbol{u}$ by

$$\boldsymbol{u} = \frac{\partial \boldsymbol{\varepsilon}}{\partial t}. \tag{3.35}$$

In an elastic medium, transverse waves obey the wave equation

$$\nabla^2 \boldsymbol{\varepsilon} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{\varepsilon}}{\partial t^2} = 0. \tag{3.36}$$

Because of (3.35), this is the same as (3.30). From $\operatorname{div} \boldsymbol{u} = 0$ then follows $\operatorname{div} \boldsymbol{\varepsilon} = 0$. For a transverse wave propagating into the x-direction, $\varepsilon_x = \varepsilon_1 = 0$, where the condition $\operatorname{div} \varepsilon = 0$ leads to

$$\frac{\partial \varepsilon_2}{\partial x_2} + \frac{\partial \varepsilon_3}{\partial x_3} = \varepsilon_{22} + \varepsilon_{33} = 0. \tag{3.37}$$

hence

$$\varepsilon_{33} = -\varepsilon_{22}, \tag{3.38}$$

The identity with a gravitational wave is established putting

$$2\,\varepsilon_{ik} = h_{ik}. \tag{3.39}$$

In both types of transverse waves the vortices are coupled through compression waves propagating with the velocity of light. Two parameters still remain undetermined: The average distance between the vortices and their radius of curvature. For a stable system of line vortices, as it is realized in the Karman vortex street, the average distance of separation between the vortex filaments is several 100 times larger than the vortex core radius [4]. By colliding with each other, snapping and reconnecting, the vortices of the vortex sponge are likely to settle into a lattice of vortex rings. Because the mutual disturbance between the vortices is there larger than in the two-dimensional lattice of line vortices, the distance of separation for a stable configuration is there likely to be larger as well. In a lattice of vortex rings the distance of separation between the rings should be of the same order of magnitude as the ring diameter. A lattice constant $10^3 - 10^4$ times larger than the Planck length would imply an upper cut-off of the transverse wave modes at the same length scale conjectured for the unification of the electroweak and strong interaction.

Figure 2 shows how a lattice of vortex rings would be deformed for the two types of transverse waves.

4. Charge As Zero Point Oscillations of Planck Mass Particles

Planck mass particles bound in vortex filaments or in rotons, execute zero point oscillations possessing a kinetic energy

$$|E| \sim \hbar c / r_p \tag{4.1}$$

with an energy density within the filaments

$$|\varepsilon| \sim \hbar c / r_p^4. \tag{4.2}$$

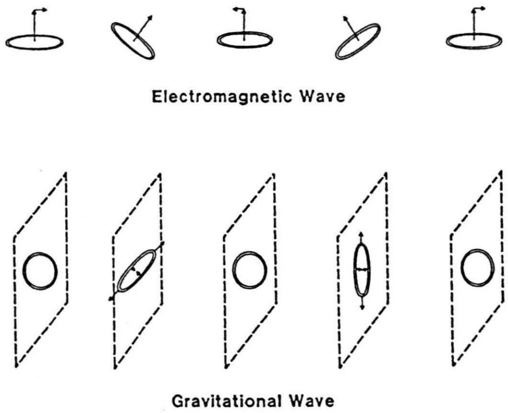


Fig. 2. Deformation of the vortex lattice for an electromagnetic and a gravitational wave.

Because this energy density is of the same order of magnitude as the energy density g^2 of a scalar gravitational field g of a Planck mass at the distance r_p,

$$g=\sqrt{G}\, m_p/r_p^2=\sqrt{\hbar c}/r_p^2, \tag{4.3}$$

it gives a simple explanation of the phenomenon of charge: It is the result of the quantum mechanical fluctuations of Planck mass particles bound in the vortex filaments, generating a virtual phonon field in the Planck mass fluid which in turn generates an attractive Newtonian force field. In support of this explanation, we note that the electromagnetic, the strong and the weak interaction are within about two orders of magnitude equal to $G m_p^2/\hbar c=1$ (like $e^2/\hbar c \simeq 1/137$, etc.). The much smaller coupling constant of the gravitational interaction of an elementary particle finds its explanation in the near complete cancellation of the zero point oscillations of the positive and negative Planck mass particles, with an elementary particle to be understood as a quasiparticle of the many body Planck mass system. With the kinetic zero point energy of the Planck mass particles acting as the source of the gravitational field, a slight imbalance in the kinetic energy of the positive and negative Planck mass fluid permits a gravitational coupling constant arbitrarily smaller than $G m_p^2/\hbar c=1$.

With all charges having their origin in the quantum mechanical zero point oscillations of the Planck mass particles, and with the vacuum occupied by an equal number of positive and negative Planck mass particles, the average value of the zero point energy of the vacuum is zero and with it the cosmological constant. And with the positive energy exactly balanced by the negative gravitational energy, the critical mass density parameter Ω is equal to one. With the phenomenon of charge reduced to the zero point oszillations of Planck masses bound in vortex filaments, and with an equal number of positive and negative vortex filaments to keep the total vacuum energy equal to zero, the sum of all charges must vanish. This is confirmed to a high degree of accuracy for the electric and color charges, but also for all the gravitational charges whereby the cosmological constant becomes equal to zero.

5. Dirac Spinors

From the vortex resonance (3.15) follows the mass

$$m_v^{\pm} \simeq \pm m_p (r_p/R)^2 \tag{5.1}$$

(for $R/r_p \sim 10^3-10^4$ it would have an energy $m_v^{\pm} c^2 \simeq \pm 10^{12}$ GeV). In a vortex lattice it can move like an excitonic quasiparticle with a double positive-negative mass vortex. Two mass dipole quasiparticle configurations are possible, one where the vortices are corotating ant the other one where they are counter-rotating. Through to the zero point oscillations of the Planck mass particles bound in the vortex filaments, the positive and negative mass components of the mass dipole interact gravitationally by a scalar Newtonian gravitational potential.

The gravitational interaction energy of a positive with a negative mass is always positive, resulting in a small residual positive mass of the two gravitationally interacting positive-negative mass resonances (5.1). The existence of such a residual mass has its cause in the nonlinearity of the gravitational interaction and explains why there can be no negative mass quasiparticles of this kind.

Adding the mass m of the small positive gravitational interaction energy to m_v^+, whereby $m^+ = m_v^+ + m$, and $m^- = m_v^-$, one obtains a configuration, which has been called a pole-dipole particle. Its importance is that it can simulate Dirac spinors [5]. With $m^+ > |m^-|$, one has $m^+ - |m^-| \ll m^+$. The center of mass of this two body configuration is still on the line connecting m^+ with m^-, but not located in between m^+ and m^- (see Figure 3). If m^+ is separated by the distance r from m^-, and if the distance of m^+ from the center of mass is r_c then because of $m \ll m^+ \simeq |m^-|$, $r \ll r_c$.

Conservation of the center of mass requires that

$$m^+ r_c = |m^-| (r_c + r). \tag{5.2}$$

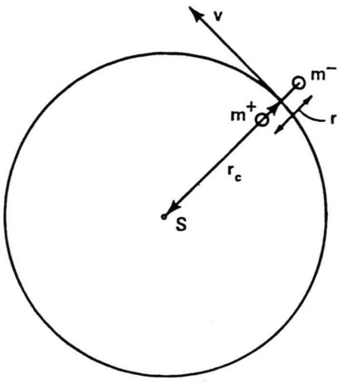


Fig. 3. A pole-dipole particle executes a circular motion around its center of mass S.

The angular momentum of the pole-dipole particle is

$$J_z = [m^+ r_c^2 - |m^-| (r_c + r)^2]\,\omega, \tag{5.3}$$

where ω is the angular velocity around the center of mass. With $m = m^+ - |m^-|$ and $p = m^+ r \simeq |m^-| r \simeq m r_c$, where m is the mass pole and p the mass dipole and with p directed from m^+ to m^-, one finds with the help of (5.2) that

$$J_z = -m^+ r\, r_c \omega = -p\, v. \tag{5.4}$$

In the limit $v \to c$ one has

$$J_z = -m\, c\, r_c. \tag{5.5}$$

The angular momentum is negative because m^- is separated by a larger distance from the center of mass than m^+.

Applying the solution of the well-known nonrelativistic quantum mechanical two-body problem with Coulomb interaction to the pole-dipole particle with Newtonian interaction, one can obtain an expression for m. For the Coulomb interaction, the groundstate energy is

$$W_0 = -\frac{1}{2}\frac{m^* e^4}{\hbar^2}, \tag{5.6}$$

where m^* is the reduced mass of the two-body system, with the potential energy $-e^2/r$ for two charges $\pm e$, of opposite sign. By comparison, the gravitational potential energy of two masses of opposite sign is $+G m^+ |m^-|/r \simeq +G |m_v^{\pm}|^2/r$ instead, and one thus has to make the substitution $e^2 \to -G |m_v^{\pm}|^2$. The reduced mass is

$$\frac{1}{m^*} = \frac{1}{m^+} + \frac{1}{m^-} = \frac{1}{m^+} - \frac{1}{|m^-|} \simeq -\frac{m}{|m_v^{\pm}|^2}, \tag{5.7}$$

and by putting $W_0 = m\, c^2$, one finds from (5.6) that

$$m = (1/\sqrt{2})\, |m_v^{\pm}|^3 / m_p^2. \tag{5.8}$$

Because of (5.1), this is

$$m/m_p = (1/\sqrt{2})\, (r_p/R)^6. \tag{5.9}$$

The Bohr radius for the hydrogen atom is

$$r_B = \hbar^2 / m^* e^2, \tag{5.10}$$

which by the substitution for e^2 and m^*, becomes

$$r_v = \hbar/\sqrt{2}\, |m_v^{\pm}|\, c = (r_p/\sqrt{2})\, (R/r_p)^2. \tag{5.11}$$

Equating m with the electron mass, one obtains $R/r_p \simeq 5\,000$, in fairly good agreement with what can be expected from the hydrodynamic stability of a three-dimensional vortex lattice. One also finds that $r_v \sim 10^{-26}$ cm, for the size of an electron. (No other theory can explain the very small number $m/m_p \sim 10^{-22}$).

With the gravitational interaction of two counter-rotating masses reduced by the factor $\gamma^{-2} = 1 - v^2/c^2$, where v is the rotational velocity [6], the mass obtained from the counterrotating vortices is much smaller. By order of magnitude one should have $\gamma\, m_v\, R\, c \sim \hbar$, and hence $\gamma \sim R/r_p$, whereby instead of (5.9) one has

$$m/m_p \sim (r_p/R)^8. \tag{5.12}$$

For $R/r_p \simeq 5 \times 10^3$, this leads to a mass of $\sim 2 \times 10^{-2}$ eV, making it a possible candidate for the neutrino mass.

The result expressed by (5.9) amounts to a computation of the renormalization constant, which in this theory is finite.

In the pole-dipole particle configuration, the spin angular momentum is the orbital angular momentum of the motion around the center of mass located on the line connecting m^+ with m^-. The rules of quantum mechanics permit radial s-wave oszillations of m^+ against m^-. For the nonrelativistic pole-dipole particle configuration, they lead to the correct angular momentum quantization as can be seen as follows: From Bohr's angular momentum quantization rule $m^* r_v\, v = \hbar$, where $v = r_v\, \omega$, one obtains by inserting the values for m^* and r_v that $m c^2 = -(1/2)\, \hbar\, \omega$, and because of $\omega = c/r_c$ that $m\, r_c\, c = -(1/2)\, \hbar$. Inserting this into (5.5) leads to $J_z = (1/2)\, \hbar$. Therefore, even in the nonrelativistic limit, the correct angular momentum quantization rule is obtained, the only one consistent with Dirac's relativistic wave equation. For the mutual oscillating velocity, one finds $v/c=(|m_v^{\pm}|/m_p)^2 = r_p/R)^4 \ll 1$, showing that our nonrelativistic approximation appears quite well justified.

The presence of negative masses leads to a "Zitterbewegung", by which a positive mass is accelerated. As it was shown by Bopp [7], the presence of negative masses can be accounted for in a generalized dynamics, in which the Lagrange function also depends on the acceleration. The equations of motion are there derived from the variational principle

$$\delta \int L(q_k, \dot{q}_k, \ddot{q}_k)\, dt = 0 \tag{5.13}$$

or from

$$\delta \int \Lambda(x_\alpha, u_\alpha, \dot{u}_\alpha)\, ds = 0, \tag{5.14}$$

where $u_\alpha = dx_\alpha/ds$, $\dot{u}_\alpha = du_\alpha/ds$, $ds=(1-\beta^2)^{1/2}\, dt$, $\beta = v/c$, $x_\alpha=(x_1, x_2, x_3, ict)$, and where $L=\Lambda(1-\beta^2)^{1/2}$. With the subsidiary condition $F=u_\alpha^2=-c^2$, the Euler–Lagrange equation for (5.14) is

$$\frac{d}{ds}\left(\frac{\partial(\Lambda+\lambda F)}{\partial u_\alpha} - \frac{d}{ds}\frac{\partial \Lambda}{\partial \dot{u}_\alpha}\right) - \frac{\partial \Lambda}{\partial x_\alpha} = 0 \tag{5.15}$$

with a Lagrange multiplier λ. In the absence of external forces, Λ can only depend on $\dot{u}_\alpha^2$. The most simple assumption is a linear dependence

$$\Lambda = -k_0 - (1/2)\, k_1\, \dot{u}_\alpha^2, \tag{5.16}$$

whereby (5.15) becomes

$$\frac{d}{ds}(2\lambda\, u_\alpha + k_1\, \ddot{u}_\alpha) = 0 \tag{5.17}$$

or

$$2\dot{\lambda}\, u_\alpha + 2\lambda\, \dot{u}_\alpha + k_1\, \dddot{u}_\alpha = 0. \tag{5.18}$$

Differentiating the subsidiary condition, one has

$$u_\alpha \dot{u}_\alpha = 0, \quad u_\alpha \ddot{u}_\alpha + \dot{u}_\alpha^2 = 0, \quad u_\alpha \dddot{u} + 3\dot{u}_\alpha \ddot{u}_\alpha = 0, \tag{5.19}$$

by which (5.18) becomes

$$-2\dot{\lambda} - 3k_1\, \dot{u}_\alpha \ddot{u}_\alpha = -2\dot{\lambda} - \frac{3}{2} k_1 \frac{d}{ds}(\dot{u}_\alpha^2) = 0. \tag{5.20}$$

It has the integral (summation over v)

$$2\lambda = k_0 - (3/2)\, k_1\, \dot{u}_v^2, \tag{5.21}$$

where k_0 appears as a constant of integration. By inserting (5.21) into (5.17) the Lagrange multiplier is eliminated and one has

$$\frac{d}{ds}\left[\left(k_0 - \frac{3}{2}k_1\, \dot{u}_v^2\right) u_\alpha + k_1\, \ddot{u}_\alpha\right] = 0. \tag{5.22}$$

That this is the equation of motion of a pole-dipole particle can be shown by writing it as follows:

$$\frac{d\boldsymbol{P}_\alpha}{ds} = 0, \quad \boldsymbol{P}_\alpha = \left(k_0 - \frac{3}{2}k_1\, \dot{u}_v^2\right) u_\alpha + k_1\, \ddot{u}_\alpha. \tag{5.23}$$

As can be seen by the angular momentum conservation law

$$\frac{dJ_{\alpha\beta}}{ds} = 0, \tag{5.24}$$

where

$$J_{\alpha\beta} = [\boldsymbol{x}, \boldsymbol{P}]_{\alpha\beta} + [\boldsymbol{u}, \boldsymbol{p}]_{\alpha\beta}, \tag{5.25}$$

the mass dipole moment is

$$p_\alpha = -k_1\, \dot{u}_\alpha. \tag{5.26}$$

For a particle at rest with $P_k=0$, $k=1, 2, 3$, one obtains

$$J_{k\ell} = [\boldsymbol{u}, \boldsymbol{p}]_{k\ell} = u_k\, p_\ell - u_\ell\, p_k, \tag{5.27}$$

which is the spin angular momentum.

The energy of a pole-dipole particle at rest, for which $u_4 = ic\gamma$, is determined by the fourth component of (5.23):

$$P_4 = im = i\left(k_0 - \frac{3}{2}k_1\, \dot{u}_v^2\right) c\gamma. \tag{5.28}$$

From (5.23) and (5.24) for $P_k=0$, $k=1, 2, 3$, one obtains for the dipole moment

$$p = \left(k_0 - \frac{3}{2}k_1\, \dot{u}_v^2\right) r_c. \tag{5.29}$$

Because of (5.28) one has

$$p = m\, r_c/\gamma. \tag{5.30}$$

With $\boldsymbol{u} = \gamma\, \boldsymbol{v}$, one obtains for the spin angular momentum

$$J_z = -p\,u = -m\,v\,r_c \simeq -m\,c\,r_c, \tag{5.31}$$

which is the same as (5.5).

For the transition to wave mechanics, one needs the equation of motion in canonical form. From $L=\Lambda\, ds/dt$, one obtains by separating space and time parts (using units in which $c=1$):

$$L=-\left(k_0+\frac{1}{2}k_1\,\dot{u}_\alpha^2\right)(1-v^2)^{1/2},$$

$$\dot{u}_\alpha^2=\frac{1}{[(1-v^2)^{1/2}]^4}\left[\dot{\boldsymbol{v}}^2+\left(\frac{\boldsymbol{v}\cdot\dot{\boldsymbol{v}}}{(1-v^2)^{1/2}}\right)^2\right], \tag{5.32}$$

where $L=L(\boldsymbol{r},\dot{\boldsymbol{r}},\ddot{\boldsymbol{r}})$. From

$$\boldsymbol{P}=\frac{\partial L}{\partial \boldsymbol{v}}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\boldsymbol{v}}},\qquad \boldsymbol{s}=\frac{\partial L}{\partial \dot{\boldsymbol{v}}} \tag{5.33}$$

one has to compute the Hamilton function

$$H=\boldsymbol{v}\cdot\boldsymbol{P}+\dot{\boldsymbol{v}}\cdot\boldsymbol{s}-L. \tag{5.34}$$

From $\boldsymbol{s}=\partial L/\partial\dot{\boldsymbol{v}}$ one obtains

$$\boldsymbol{s}=-\frac{k_1}{\sqrt{1-v^2}^{\,3}}\left[\dot{\boldsymbol{v}}+\frac{(\boldsymbol{v}\cdot\dot{\boldsymbol{v}})\,\boldsymbol{v}}{1-v^2}\right],$$

$$\dot{\boldsymbol{v}}=-\frac{\sqrt{1-v^2}^{\,3}}{k_1}\,[\boldsymbol{s}-(\boldsymbol{v}\cdot\boldsymbol{s})\,\boldsymbol{v}], \tag{5.35}$$

by which with the help of (5.32) $\dot{\boldsymbol{v}}\cdot\boldsymbol{s}$ can be expressed in terms of $\boldsymbol{v}$ and $\boldsymbol{s}$. In these variables, the angular momentum conservation law (5.24) assumes the form

$$\boldsymbol{r}\times\boldsymbol{P}+\boldsymbol{v}\times\boldsymbol{s}=\text{const} \tag{5.36}$$

with the vector $\boldsymbol{s}$ equal the dipole moment. For the Hamilton function (5.34) one then finds

$$H=\boldsymbol{v}\cdot\boldsymbol{P}+k_0(1-v^2)^{1/2}$$
$$-(1/2\,k_1)\,(1-v^2)^{3/2}\,[s^2-(\boldsymbol{s}\cdot\boldsymbol{v})^2]. \tag{5.37}$$

Putting

$$\boldsymbol{P}=\frac{\hbar}{i}\frac{\partial}{\partial\boldsymbol{r}},\qquad \boldsymbol{v}=\boldsymbol{\alpha},\qquad (1-v^2)^{1/2}=\alpha_4, \tag{5.38}$$

where $\alpha=\{\boldsymbol{\alpha},\alpha_4\}$ are the Dirac matrices, one finally obtains the Dirac equation

$$\frac{\hbar}{i}\frac{\partial\psi}{\partial t}+H\,\psi=0, \tag{5.39}$$

where

$$H=\alpha_1\,P_1+\alpha_2\,P_2+\alpha_3\,P_3+\alpha_4\,m,$$
$$\alpha_\mu\,\alpha_\nu+\alpha_\nu\,\alpha_\mu=2\,\delta_{\mu\nu}, \tag{5.40}$$

with the mass given by

$$m=k_0-(1/2\,k_1)\,(1-v^2)\,[s^2-(\boldsymbol{s}\cdot\boldsymbol{v})^2]. \tag{5.41}$$

Higher particle families result from by internal excited states of the pole-dipole configuration, to be obtained by Bopp's generalized mechanics [7]. More generally one may replace (5.16) by

$$\Lambda=-f(Q),\qquad Q=\dot{u}_\alpha^2, \tag{5.42}$$

where $f(Q)$ is an arbitrary function of Q which depends on the internal structure of the mass dipole. With (5.42) one has for (5.15)

$$\frac{\mathrm{d}}{\mathrm{d}s}\left\{[f(Q)-4\,Q\,f'(Q)]\,u_\alpha+2\frac{\mathrm{d}}{\mathrm{d}s}[f'(Q)\,\dot{u}_\alpha]\right\}=0, \tag{5.43}$$

where

$$p_\alpha=-2\,f'(Q)\,\frac{\mathrm{d}u_\alpha}{\mathrm{d}s} \tag{5.44}$$

is the dipole moment. For the simple pole-dipole particle one has according to (5.16) $f(Q)=k_0+(1/2)\,k_1\,Q$, whereby $p_\alpha=-k_1\,\dot{u}_\alpha$ as in (5.26).

Instead of (5.35) one now has

$$\boldsymbol{s}=-2\frac{f'(Q)}{\sqrt{1-v^2}^{\,3}}\left[\dot{\boldsymbol{v}}+\frac{(\boldsymbol{v}\cdot\dot{\boldsymbol{v}})\,\boldsymbol{v}}{1-v^2}\right],$$

$$\dot{\boldsymbol{v}}=-\frac{1}{2}\frac{\sqrt{1-v^2}^{\,3}}{f'(Q)}\,[\boldsymbol{s}-(\boldsymbol{v}\cdot\boldsymbol{s})\,\boldsymbol{v}]. \tag{5.45}$$

Computation of $\dot{\boldsymbol{v}}\cdot\boldsymbol{s}$ from both of these equations leads to the identity

$$4\,Q\,f'(Q)^2=R=(1-v^2)\,[s^2-(\boldsymbol{v}\cdot\boldsymbol{s})^2], \tag{5.46}$$

from which the function $Q=Q(R)$ can be obtained, and by which $\dot{\boldsymbol{v}}$ can be eliminated from H:

$$H=\boldsymbol{v}\cdot\boldsymbol{P}+\dot{\boldsymbol{v}}\cdot\boldsymbol{s}-L=\boldsymbol{v}\cdot\boldsymbol{P}+\sqrt{1-v^2}\,F(R), \tag{5.47}$$

where

$$F(R)=f(Q)-2\,Q\,f'(Q). \tag{5.48}$$

For the wave mechanical treatment of this problem it is convenient to use the four-dimensional representation by making a canonical transformation from the variables $(\boldsymbol{v},v_0,\boldsymbol{s},s_0)$ to (u_α,p_α):

$$\boldsymbol{s}\cdot\mathrm{d}\boldsymbol{v}+s_0\,\mathrm{d}v_0+u_\alpha\,\mathrm{d}p_\alpha=\mathrm{d}\Phi(\boldsymbol{v},v_0,p_\alpha) \tag{5.49}$$

with the generating function

$$\Phi=\frac{v_0}{\sqrt{1-v^2}}(\boldsymbol{v}\cdot\boldsymbol{p}+i\,p_4), \tag{5.50}$$

and where v_0, θ_0 are superfluous coordinates. Expressed in the new variables, one has

$$R=-\frac{1}{2}M_{\alpha\beta}^2,\qquad M_{\alpha\beta}=u_\alpha\,p_\beta-u_\beta\,p_\alpha. \tag{5.51}$$

With $P_\alpha = \{\boldsymbol{P}, i H\}$, (5.47) is replaced by

$$K = u_\alpha P_\alpha + \sqrt{-u_\alpha^2}\, F(R) = 0. \tag{5.52}$$

Because $u_\alpha^2 = -1$ and $u_\alpha p_\alpha = 0$, the superfluous coordinates v_0 and s_0 can be eliminated. Putting

$$P_\alpha = \frac{\hbar}{i}\frac{\partial}{\partial x_\alpha}, \quad p_\alpha = \frac{\hbar}{i}\frac{\partial}{\partial u_\alpha}, \tag{5.53}$$

one obtains the wave equation

$$K\psi \equiv \left[\left(u_\alpha, \frac{\hbar}{i}\frac{\partial}{\partial x_a}\right) + F(R)\right]\psi(x, u) = 0, \tag{5.54}$$

where

$$R = -\frac{1}{2} M_{\alpha\beta}^2,$$

$$M_{\alpha\beta} = \frac{\hbar}{i}\left(u_\alpha \frac{\partial}{\partial u_\beta} - u_\beta \frac{\partial}{\partial u_\alpha}\right). \tag{5.55}$$

For $\boldsymbol{P} = 0$, the wave function has the form

$$\psi(x, u) = \psi(u)\, e^{-i\varepsilon t/\hbar} \tag{5.56}$$

with the wave equation for $\psi(u)$:

$$F(R)\,\psi(u) = \frac{\varepsilon}{\sqrt{1-v^2}}\,\psi(u), \tag{5.57}$$

or if G is the inverse function for F:

$$R\,\psi(u) = G\left(\frac{\varepsilon}{\sqrt{1-v^2}}\right)\psi(u). \tag{5.58}$$

From the condition $u_\alpha p_\alpha = 0$ follows that

$$R = -\hbar^2 \frac{\partial^2}{\partial u_\alpha^2}. \tag{5.59}$$

With (θ, ϕ spherical polar coordinates)

$$u_\alpha = [\sinh\alpha \cdot \sin\theta \cdot \cos\phi,\ \sinh\alpha \cdot \sin\theta \cdot \sin\phi,\ \sinh\alpha \cdot \cos\theta,\ i\cosh\alpha],$$

$$\psi = \psi_0 / \sinh\alpha, \quad \tanh\alpha = v \tag{5.60}$$

the wave equation becomes

$$-\left[\frac{\partial^2}{\partial\alpha^2} - 1 - \frac{M^2}{\sinh^2\alpha}\right]\psi_0 = G(\varepsilon\cosh\alpha)\,\psi_0, \tag{5.61}$$

where

$$M^2 = (\boldsymbol{v}\times\boldsymbol{s})^2 = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}, \tag{5.62}$$

having the eigenvalues $j(j+1)$, where j is an integer. The wave equation, therefore, finally becomes

$$\frac{d^2\psi_0}{d\alpha^2} = V(\alpha)\,\psi_0 = \left[1 + \frac{j(j+1)}{\sinh^2\alpha} - G(\varepsilon\cosh\alpha)\right]\psi_0. \tag{5.63}$$

The eigenvalues can be obtained by the WKB method, with the factor $j(j+1)$ be replaced by $\left(j+\frac{1}{2}\right)^2$ to account for the singularity at $\alpha = 0$. The eigenvalues are then determined by the equation

$$J = \frac{1}{\pi}\int_{\alpha_1}^{\alpha_2} \sqrt{-V(\alpha)}\, d\alpha = n + \frac{1}{2}, \quad n = 0, 1, 2, \dots \tag{5.64}$$

with

$$V(\alpha) = 1 + \frac{(j+\frac{1}{2})^2}{\sinh^2\alpha} - G(\varepsilon\cosh\alpha). \tag{5.65}$$

Of special interest are the cases where $j = -1/2$, because they correspond to the correct angular momentum quantization rule for the Zitterbewegung. For $j = -1/2$ one simply has

$$V(\alpha) = 1 - G(\varepsilon\cosh\alpha). \tag{5.66}$$

To obtain an eigenvalue requires a finite value of the phase integral (5.64). The function $G(x)$, ($x = \varepsilon\cosh\alpha$), must therefore qualitatively have the form of a parabola cutting the line $G = 1$ at two points x_1, x_2 in between which $G(x) > 1$. One can then distinguish two limiting cases. First if $\varepsilon \ll 1$ and second if $\varepsilon \gg 1$. In both cases one may approximate (5.64) as follows:

$$J \simeq (1/\pi)\,\overline{\sqrt{-V(\alpha)}}\,(\alpha_2 - \alpha_1). \tag{5.67}$$

In the first case $\alpha \gg 1$ and one has

$$\alpha \simeq \ln\left(\frac{x}{\varepsilon} + \sqrt{\frac{x^2}{\varepsilon^2} - 1}\right) \approx \ln\left(\frac{2x}{\varepsilon}\right) - \frac{\varepsilon^2}{4x^2} + \dots, \tag{5.68}$$

hence

$$\alpha_2 - \alpha_1 \simeq \ln\left(\frac{x_2}{x_1}\right) + \frac{x_2^2 - x_1^2}{x_1^2 x_2^2}\frac{\varepsilon^2}{4} + \dots. \tag{5.69}$$

In the second case one has

$$\alpha \simeq \sqrt{2}\left(\frac{x}{\varepsilon} - 1\right)^{1/2} \tag{5.70}$$

or if $x/\varepsilon \gg 1$ simply

$$\alpha \simeq \sqrt{2x/\varepsilon}, \tag{5.71}$$

hence

$$\alpha_2-\alpha_1 \simeq \sqrt{\frac{2}{\varepsilon}}\,(x_2^{1/2}-x_1^{1/2}). \tag{5.72}$$

One, therefore, sees that the phase integral has for $\varepsilon \ll 1$ the form $J=a+b\,\varepsilon^2$, but for $\varepsilon \gg 1$ the form $J=a/\sqrt{\varepsilon}$. The $J(\varepsilon)$ curve can for this reason cut twice the lines $J=1/2$ $(n=0)$, and $J=3/2$ $(n=1)$. This means that there is only a finite number of excited states for the pole-dipole configuration, and hence a finite number of particle families. Adjusting the phase integral to the first three families, one concludes that there is at most one more family.

The finiteness of the number of particle families is here a consequence of Bopp's nonlinear dynamics involving negative masses, not as in superstring theories where it results from a topological constraint.

6. Quark-Lepton Symmetries

More difficult is the interpretation of the quark-lepton symmetries, but they can possibly be explained like the fractional quantum Hall effect in condensed matter physics, as a fluid of quasiparticles with fractional charges. The fractionally charged particles appearing in the anomalous quantum Hall effect are, of course, quasiparticles. It is for this reason plausible that the fractionally charged quarks might be understood along the same lines of thought as the fractionally charged electrons in thin layers of condensed matter.

As Laughlin [8] has shown, an electron gas confined within a thin sheet can be described by the wave function

$$\psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots \boldsymbol{r}_N)=\left[\prod_{j<k}(z_j-z_k)^{\mu}\right]\left[\prod_j e^{-|z_j|^2/4\ell^2}\right] \tag{6.1}$$

where $z_j=x_j-i\,y_j$ is the coordinate of the j^{th} electron in complex notation and $\ell^2=\hbar c/eH$ (obtained from $m\ell^2\,\omega=\hbar$ and $\omega=e\,H/mc$ for the lowest Landau level.) The magnetic field H is directed perpendicular to the sheet. If μ is an odd integer, the wave function is completely antisymmetric, obeying Fermi statistics, made up from states of the first Landau level with the kinetic energy equal to $(1/2)\,\hbar\omega$ per electron. For the square of the wave function one has

$$|\psi|^2=e^{-\beta H},$$
$$\beta H=2\mu\sum_{j<k}\ell n\,|\boldsymbol{r}_j-\boldsymbol{r}_k|+(1/2\ell^2)\sum_j|\boldsymbol{r}_j|^2, \tag{6.2}$$

which is the probability distribution $|\psi|^2$ of a one-component two-dimensional plasma.

For $\mu=1$, the wave function is a Slater determinant, but this wave function does not describe the situation actually observed. Numerical calculations for four to six electrons done by Laughlin, rather, show that the wave function (6.1) for $\mu=3$ gives a much better agreement. For this wave function, which satisfactorily explains the fractional quantized Hall effect, plateaus in the conductivity are found to occur in multiple steps of $(1/3)\,e^2/h$. For the quasiparticle interpretation of this wave function, one keeps all electrons, except one, fixed in their position and carries out a closed loop motion of the one electron around a point at which the wave function vanishes. This displacement produces the phase shift

$$\Delta\phi=(e/\hbar c)\oint \boldsymbol{A}\cdot d\boldsymbol{s}=(e/\hbar c)\int \boldsymbol{H}\cdot d\boldsymbol{f}, \tag{6.3}$$

where $\boldsymbol{H}=\operatorname{curl}\boldsymbol{A}$. Accordingly, there should be

$$Z=(e/\hbar c)\int \boldsymbol{H}\cdot d\boldsymbol{f} \tag{6.4}$$

quasiparticles within the area $\int d\boldsymbol{f}$. To satisfy the Pauli principle there must be at least one quasiparticle at the position of each electron. In Laughlin's wave function there are exactly μ quasiparticles for each electron. We therefore have to put $Z=\mu$. The fractional quantized Hall effect then simply means that the charge of one quasiparticle is $e/3$ provided $\mu=3$. It follows that in the two-dimensional electron fluid each electron splits into three quasiparticles of charge $e/3$.

In the fluid dynamic picture of quantum mechanics, these quasiparticles can be understood as vortices into which the electron wave function splits. The Madelung transformation for the nonrelativistic Schrödinger equation of an electron in a magnetic field

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}\left(i\hbar\nabla-\frac{e}{c}\boldsymbol{A}\right)^2\psi \tag{6.5}$$

leads to the Euler equation with quantum potential

$$\frac{d\boldsymbol{v}}{dt}=-\frac{1}{m}\operatorname{grad}Q+\frac{e}{mc}\boldsymbol{v}\times\boldsymbol{H}. \tag{6.6}$$

From the Madelung transformation follows that $(\nu=0, \pm1\pm2,\ldots)$:

$$\oint \boldsymbol{v}\cdot d\boldsymbol{s}=\frac{h}{m}\nu-\frac{e}{mc}\int \boldsymbol{H}\cdot d\boldsymbol{f}, \tag{6.7}$$

which shows that the presence of a magnetic field causes the occurrence of the vortices in the electron fluid. And because of (6.3) with $Z=3$, it shows that the electron wave function splits into three vortices, each with a charge $e/3$. Variational calculations made by Laughlin show that the wave function is best described by an equilateral triangular vortex lattice, very much as in classical fluid dynamics, where in the Karman vortex street a triangular arrangement of the vortices has the greatest stability.

If a magnetic field is adiabatically applied to the electron fluid, the Helmholtz theorem

$$\frac{\mathrm{d}}{\mathrm{d}t}\oint \boldsymbol{v}\cdot \mathrm{d}\boldsymbol{s}=0 \tag{6.8}$$

states that, if the circulation $\oint \boldsymbol{v}\cdot\mathrm{d}\boldsymbol{s}$ is zero before a magnetic field is applied, it remains zero thereafter. This, of course, does not imply that the circulation inside the contour taken in (6.7) cannot differ from zero, because the circulation of different vortices can add up to zero, as it would be the case for four vortices with equal and opposite circulation. A vortex configuration with the total angular momentum $(1/2)\,\hbar$ could be constructed from two vortices with opposite circulation quantum number $\nu=\pm 1$, and one vortex with $\nu=1$ with the spin quantum numbers adding up to a total angular momentum $(1/2)\,\hbar$.

To explain the fractionally charged quarks, we propose that they result from the splitting up of the electron- and neutrino-wave functions into vortices, with the splitting up caused by the weak interaction magnetic field of the WSG theory. This field acts within a thin sheet, with the vortices perpendicular to and confined within this sheet and with a minimum of three vortices needed to define the orientation of a planar sheet.

If the electron- and neutrino-wave functions split up in a similar way as it happens in the fractionally quantized Hall effect, the quark-lepton symmetries can easily be understood. The angular momentum of a vortex in units of $\hbar$ is equal to the circulation quantum number ν. The vortices for which $\nu=1$ we call A, those for which $\nu=0$ we call B, and finally those for which $\nu=-1$ we call C. The neutrino (ν) and positron (e_+) wave functions are then to be represented by six vortex states, with the lower indices giving the value for the electric charge of these vortex states:

$$(\nu)=\begin{cases}A_0\\ B_0\\ C_0\end{cases},\quad (e_+)=\begin{cases}A_{1/3}\\ B_{1/3}\\ C_{1/3}\end{cases}. \tag{6.9}$$

A second set of six vortex states is obtained by replacing the neutrino and positron by their antiparticles. We claim that the first six vortex states can reproduce all the six u and d quarks of the first family. How the neutrino and positron are composed of these vortex states is already shown in (6.9). With the index r, g, b (red, green, blue) identifying what is called the color, we have for the three colors of the u quark

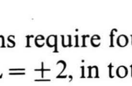
$$u_{\mathrm{r}}=\begin{cases}A_{1/3}\\ B_{1/3}\\ C_0\end{cases},\quad u_{\mathrm{g}}=\begin{cases}A_{1/3}\\ B_0\\ C_{1/3}\end{cases},\quad u_{\mathrm{b}}=\begin{cases}A_0\\ B_{1/3}\\ C_{1/3}\end{cases}. \tag{6.10}$$

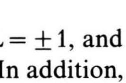
For the three d quarks we have

$$d_{\mathrm{r}}=\begin{cases}A_0\\ B_0\\ C_{-1/3}\end{cases},\quad d_{\mathrm{g}}=\begin{cases}A_0\\ B_{-1/3}\\ C_0\end{cases},\quad d_{\mathrm{b}}=\begin{cases}A_{-1/3}\\ B_0\\ C_0\end{cases}. \tag{6.11}$$

Because the vortices are substates of the leptons, color confinement then simply means that only those vortex configurations which can be combined into leptons are able to assume the form of free particles. Mesons are made up from quark-antiquark configurations, each containing three vortices and three antivortices.

If the vortices interact, they do this by the exchange bosons. But because they are confined within a thin sheet, the bosons are massive, very much as an electromagnetic wave in a wave guide where the photons are massive and have a longitudinal component in addition to their transverse component. It is then possible to explain the eight gluons of the standard model. The gluons are bosons transmitting angular momentum. To change an A vortex into a B vortex, a B vortex into a C vortex, or vice versa, requires a change in the angular momentum $\Delta L=\pm 1$, and to change an A vortex into a C vortex, or vice versa, a change by $\Delta L=\pm 2$ is needed. These changes can be made by just two angular momentum operators with $L=1$ and $L=2$, having $\sum(2L+1)=2+1+4+1=8$ states, equal to the number of the gluons in QCD. The transitions in QCD, identified by red-green (r-g), red-blue (r-b), and green-blue (g-b), lead to changes in the angular momentum of the vortices in the following way:

r → g = g → b : 

, r → b : 

These transitions require four changes $\Delta L=\pm 1$, and two changes $\Delta L=\pm 2$, in total 6 changes. In addition, there is the change for which $\Delta L = 0$

$$r \to r = g \to g = b \to b: \begin{matrix} A \to A \\ B \to B \\ C \to C \end{matrix}, \tag{6.13}$$

realized with the two $L_z = 0$ – components of the angular momentum operators for $L = 1$ and $L = 2$, which for this reason must be counted twice. Together with the 6 changes involving $\Delta L = \pm 1, \pm 2$ one has a total of 8 possible changes involving the exchange of angular momentum. The eight gluons of QCD are not to be identified with these eight angular momentum transmitting bosons, but rather with certain combinations of them. A color-changing gluon would be always a superposition of a spin 1 and a spin 2 boson. A transition leaving the color unchanged would involve the superposition of spin 2 or spin 1 bosons. The color charge is thereby reduced to angular momentum, and through angular momentum quantization to the zero point fluctuations of the Planck masses like the other charges.

The same decomposition into vortex states done here for the first family can be repeated for the higher generations. And the weak interaction phenomenon is explained by the exchange of bosons made up from spin 1 and spin 2 angular momentum transitions in between the vortices of the leptons given by (6.9).

We remark that our model can be compared with the rishon model, with the rishons turning out to be vortex states. The three hypercolor charges of the rishon model are the three angular momentum states $L = 1, 0, -1$ of the vortices. The prescription of the rishon model that only those configuration are possible which are color neutral with regard to the hypercolor, is explained by the requirement that the vortex states must add up to zero angular momentum.

For the splitting up of the electron- and neutrino wave functions the weak interaction requires that the magnetic field H_w must satisfy the inequality

$$e H_w \delta \gg m_n c^2, \tag{6.14}$$

where m_n is the nucleon mass and δ the thickness of the sheet confining the vortices. The mass of the intermediate vector boson is related to the weak interaction magnetic vector potential A_w (with the vacuum gauge $A_w = 0$) by

$$m_w c^2 = e A_w . \tag{6.15}$$

Because $A_w = H_w \cdot \delta$ and $m_w \delta c \simeq \hbar$, one obtains from (6.15)

$$H_w \simeq (m_w c^2)^2 / e \hbar c \simeq 10^{26} \text{ Gauss}. \tag{6.16}$$

With $m_w \gg m_n$, the inequality (6.14) is satisfied.

The intermediate vector bosons of the WSG theory are in the model explained as vortex-antivortex pairs from the first generation. This implies the existence of heavier intermediate vector bosons made up from vortices of the higher generations. Assuming that the mass ratios of the vector bosons are about equal to the mass ratios of the leptons, the vector boson of the second generation should have a mass of $\sim 2 \times 10^4$ GeV.

7. Parity and CP Violation

One of the most puzzling aspects of high energy physics is the phenomenon of parity violation in weak interaction. Shortly after its discovery it was believed that its cause is a vanishing rest mass of the neutrino. A zero rest mass neutrino sustains its helicity and nature could have selected (for reasons unknown) neutrinos of just one helicity. The solar neutrino problem, however, suggests that the neutrino has a finite, albeit very small rest mass. A zero rest mass neutrino would be also difficult to reconcile with the view that Dirac spinors are composed positive–negative mass pole-dipole particles. A close inspection of parity violation shows that it only occurs in conjunction with the W-intermediate vector boson of the WSG theory. Rather than assuming the existence of chiral left-handed zero rest mass neutrinos, the whole experimental material can be explained as well by assuming that the W-particles act like left-handed "screws" interacting only with left-handed neutrinos. If this is the cause for parity violation, the question where the W-particles get their left-handedness must be answered. The Planck aether hypothesis not only can provide a possible answer but can also explain the violation of CP invariance. All what is needed is some small imbalance in the global vorticity of the Planck aether. As in fluid dynamics it is given by $\boldsymbol{\omega} = \operatorname{curl} \boldsymbol{v}$, and for the two-component positive negative mass Planck particle superfluid by

$$\boldsymbol{\omega}_\pm = \operatorname{curl} \boldsymbol{v}_\pm . \tag{7.1}$$

For each component of the superfluid the vorticity can be quite large, even though it may be small if both are added up. With the intermediate vector bosons interacting with the positive mass component of the Planck aether, a sufficiently large global vorticity $\boldsymbol{\omega}_+ = \operatorname{curl} \boldsymbol{v}_+$ may impress its handedness on the vortex-antivortex state of which the intermediate vector boson is made up. If $\boldsymbol{\omega}_- \simeq \boldsymbol{\omega}_+$, a left small net vorticity $\boldsymbol{\omega} = \boldsymbol{\omega}_+ + \boldsymbol{\omega}_- \ll \boldsymbol{\omega}_\pm$ would break the exact matter-antimatter symmetry required to explain CP violation.

The reduction of parity and CP violation to a global vorticity implies that it may be different in other parts of the universe.

8. Lorentz Invariance

For a solid body, composed of Dirac spinor particles in static equilibrium, the electromagnetic forces acting between its positive and negative electric charges are balanced by the quantum forces. To prove Lorentz invariance as a dynamic symmetry for such bodies in static equilibrium it is sufficient to assume that the forces are derived from a scalar potential. (The proof can be trivially generalized to forces derived from a vector potential, like electromagnetic forces.)

We had shown that the Planck mass fluid has sound-like waves propagating with the velocity of light. The source of these waves are the zero point oscillations of Planck mass particles bound in vortex filaments or rotons. According to Einstein and Hopf [9] an electric charge moving with the velocity v through a stochastic field of electromagnetic radiation with a frequency-dependent energy spectrum $f(\omega)$, suffers a friction force equal to

$$F = -\text{const}\left[f(\omega) - \frac{\omega}{3}\frac{\mathrm{d}f(\omega)}{\mathrm{d}\omega}\right] v. \tag{8.1}$$

Assuming that the same also holds true for a charge generated by the zero point oszillations of a Planck mass bound in a vortex (resp. roton), the absence of a frictional force would require that

$$f(\omega) = \text{const}\; \omega^3, \tag{8.2}$$

which is the only nontrivial spectrum invariant under a Lorentz transformation. Because a vacuum filled with Planck mass particles should relax into a state with no friction, the zero point energy spectrum must be Lorentz invariant. With $4\pi\omega^2 \,\mathrm{d}\omega$ modes of oscillation in between ω and $\omega + \mathrm{d}\omega$, the energy of each mode must be proportional to ω to obtain the ω^3 dependence (8.2). Because the spectrum (8.2) is generated by collective oszillations of the discrete Planck mass particles, it must be cut off at the Planck frequency $\omega_p = c/r_p = 1/t_p$, where the zero point energy can be set equal to $(1/2)\,\hbar\,\omega_p = (1/2)\, m_p c^2$. It thus follows that the zero point energy of each mode with a frequency $\omega < \omega_p$ must be $E = (1/2)\,\hbar\,\omega$. A cut-off of the zero point energy at the Planck frequency destroys the Lorentz invariance, but only for frequencies near the Planck frequency and hence only at extremely high energies. The nonrelativistic Schrödinger equation, in which the zero point energy is expressed through the kinetic energy term $-(\hbar^2/2m)\,\nabla^2\psi$, therefore remains valid for masses $m < m_p$, to be replaced by Newtonian mechanics for masses $m > m_p$.

A cut-off at the Planck frequency generates a distinguished reference system in which the zero point energy spectrum is isotropic and in a sense at rest. In this distinguished reference system, the scalar potential from which the forces are to be derived satisfies the inhomogeneous wave equation

$$-\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} + \nabla^2 \Phi = -4\pi\varrho(\boldsymbol{r}, t), \tag{8.3}$$

where $\varrho(\boldsymbol{r}, t)$ are the sources of this field. For a body in a static equilibrium at rest in the distinguished reference system for which the sources are those of the body itself one has

$$\nabla^2 \Phi = -4\pi\varrho(\boldsymbol{r}). \tag{8.4}$$

If set into absolute motion with the velocity v along the x-axis, the coordinates of the reference system at rest with the moving body are obtained by the Galilei transformation

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t, \tag{8.5}$$

transforming (8.3) into

$$-\frac{1}{c^2}\frac{\partial^2 \Phi'}{\partial t'^2} + \left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 \Phi'}{\partial x'^2} + \frac{\partial^2 \Phi'}{\partial y'^2} + \frac{\partial^2 \Phi'}{\partial z'^2} = -4\pi\varrho'(\boldsymbol{r}', t'). \tag{8.6}$$

After the body has settled into a new equilibrium in which $\partial/\partial t' = 0$, one has instead of (8.4):

$$\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 \Phi'}{\partial x'^2} + \frac{\partial^2 \Phi'}{\partial y^2} + \frac{\partial^2 \Phi'}{\partial z^2} = -4\pi\varrho(x', y, z). \tag{8.7}$$

Comparison of (8.7) with (8.4) shows that the l.h.s. of (8.7) is the same if one sets $\Phi' = \Phi$ and $\mathrm{d}x' =$

$dx\sqrt{1-v^2/c^2}$. This implies a uniform contraction of the body by the factor $\sqrt{1-v^2/c^2}$ because the sources are contracted by the factor $\sqrt{1-v^2/c^2}$ as well, whereby the r.h.s. of (8.7) becomes equal to the r.h.s. of (8.4). Because the zero point energy is invariant under a Lorentz transformation, the quantum potential changes in the same way as Φ. The body therefore sustains its static equilibrium under a contraction by the factor $\sqrt{1-v^2/c^2}$ if set into absolute motion, explaining the Lorentz contraction dynamically.

The clock retardation effect can be derived from the contraction effect, and from there the Lorentz transformations. This original interpretation of Lorentz invariance by Lorentz and Poincaré has been worked out in every detail by Prokhovnik [10] following Builder [11].

To derive the clock retardation effect from the contraction effect one considers a light clock, which is a rod with mirrors attached to its two ends in between a light signal is sent forth and back. If the length of the rod is ℓ, and if the rod rests in the distinguished reference system, the time needed for the light signal to be sent forth and back is

$$t_0 = 2\ell/c. \tag{8.8}$$

If prior to be set into motion the rod is inclined against the x-axis by the angle φ (see Fig. 4), it appears to be inclined against the x-axis by the different angle ψ after set into motion, with ψ expressed through φ by

$$\tan\psi = \gamma\tan\varphi, \quad \gamma = (1-v^2/c^2)^{-1/2}. \tag{8.9}$$

The absolute motion then contracts the rod from ℓ to ℓ':

$$\ell' = \ell\sqrt{1-(v^2/c^2)\cos^2\varphi} = \frac{\ell}{\gamma\sqrt{1-(v^2/c^2)\sin^2\psi}}. \tag{8.10}$$

Relative to the moving rod the velocity of light is anisotropic, and for the to and fro directions given by

$$c_+ = \sqrt{c^2 - v^2\sin^2\psi} - v\cos\psi,$$
$$c_- = \sqrt{c^2 - v^2\sin^2\psi} + v\cos\psi \tag{8.11}$$

with the time t' for a to and fro signal given by

$$t' = \ell'/c_+ + \ell'/c_- = \gamma\, t_0. \tag{8.12}$$

Therefore, as seen from an observer at rest in the distinguished reference system the clock goes slower by the factor $\gamma = 1/\sqrt{1-v^2/c^2}$, independent of the inclination of the rod making up the clock. With solid bodies held together by electromagnetic forces, clocks made from solid matter should behave like light

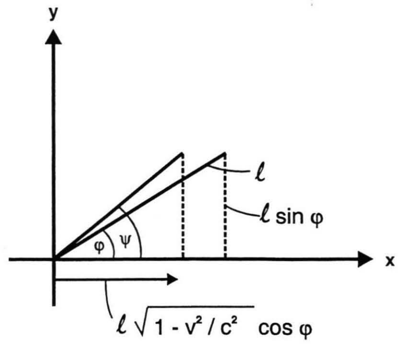


Fig. 4. The projections of a rod length ℓ along the y and x axis as seen by an observer at rest in the distinguished reference system: $\ell\sin\varphi$, $\ell\cos\varphi$ for a rod at rest and $\ell\sqrt{1-v^2/c^2}\cos\varphi$ for a rod moving with an absolute velocity v into the x-direction.

clocks. As it was claimed by Poincaré, it should for this reason be possible to obtain the Lorentz transformations solely from the contraction effect with a proper convention about the synchronization of clocks.

According to Einstein, two clocks, A and B, are synchronized if

$$t_B = \frac{1}{2}(t_A^1 + t_A^2), \tag{8.13}$$

where t_A^1 is the time a light signal is emitted from A to B, reflected at B back to A, arriving at A at the time t_A^2, and where it is *assumed* that the time t_B at which the reflection at B takes place is equal to the arithmetic average of t_A^1 and t_A^2. Only by making this assumption does the velocity of light turn out always to be isotropic and equal to c. From an absolute point of view, the following is rather true: If t_R is the absolute reflection time of the light signal at clock B, one has for the out and return journeys of the light signal from A to B and back to A, if measured by an observer in an absolute system at rest in the distinguished reference system:

$$\gamma(t_R - t_A^1) = d/c_+,$$
$$\gamma(t_A^2 - t_R) = d/c_-, \tag{8.14}$$

where d is the distance between the clocks, and where c_+ and c_- are given by (8.11). Adding the equations (8.14) one obtains

$$c(t_A^2 - t_A^1) = 2\gamma\, d\sqrt{1-(v^2/c^2)\sin^2\psi}. \tag{8.15}$$

If an observer at rest with the clock wants to measure the distance from A to B, he can measure the time it takes a light signal to go from A to B and back to A. If he assumes that the velocity of light is constant and isotropic in all inertial reference systems, including the one he is in, moving together with A and B with the absolute velocity v, this distance is

$$d' = (c/2)\,(t_A^2 - t_A^1), \tag{8.16}$$

and because of (8.15)

$$d' = \gamma\, d\,\sqrt{1-(v^2/c^2)\sin^2\psi}\,. \tag{8.17}$$

Comparing this result with (8.10), one sees that he would obtain the same distance d' if he used a contracted rod as a measuring stick, or Einstein's constant light velocity postulate. The velocity of light between A and B by using a rod to measure the distance and the time it takes a light signal in going from A to B and back to A, of course, will turn out to be equal to c, because according to (8.16)

$$\frac{2\,d'}{t_A^2 - t_A^1} = c\,. \tag{8.18}$$

Rather than using a reflected light signal to measure the distance d', the observer at A may try to measure the one-way velocity of light by first synchronizing the clock B with A and then measure the time for a light signal to go from A to B. However, since this synchronization procedure also uses reflected light signals, the result is the same. For the velocity he finds

$$\frac{d'}{t_B - t_A^1} = \frac{d'}{(1/2)\,(t_A^1 + t_A^2) - t_A^1} = \frac{2\,d'}{t_A^2 - t_A^1} = c\,. \tag{8.19}$$

By subtracting the equations (8.14) one finds that

$$t_R = t_B + (\gamma/c^2)\, v\, d \cos\psi\,, \tag{8.20}$$

which shows that from an absolute point of view the "true" reflection time t_R at clock B is only then equal to t_B if $v=0$. From an absolute point of view, the propagation of light is isotropic in the distinguished reference system only, but anisotropic in a reference system in absolute motion against the distinguished reference system. This anisotropy remains hidden due to the impossibility to measure the one way light velocity. The impossibility is expressed in the Lorentz transformations themselves, containing the scalar c^2 rather than the vector $\mathbf{c}$, through which an anisotropic light propagation would have to be expressed.

Following Prokhovnik [10], one can derive the Lorentz transformations in a way which exhibits the emergence of the relativistic effects in each step. For this to show, consider two observers, A and B, moving with uniform velocities v_A and v_B $(v_B > v_A)$ along the same straight line relative to the distinguished reference system I_s. A and B synchronize their clocks at the moment of their spatial coincidence, with the time set equal the zero I_s–time by a third clock at rest within I_s. According to (8.8) and (8.12) the times measured in I_s, I_A, and I_B are related to each other by

$$t_s = \gamma_A\, t_A = \gamma_B\, t_B\,. \tag{8.21}$$

To an observer S at rest in I_s, the relative velocity of A and B is $v_B - v_A$, with the I_s space-interval separating A and B at the time t_s equal to $(v_B - v_A)\,t_s$. The origin of the I_s reference system is chosen at a point where at the time $t_s = t_A = t_B = 0$, I_A coincides with I_B, with A's location as the origin of I_A and likewise B's location as the origin of I_B. The straight line joining the origins of I_s, I_A and I_B is chosen as the common x-axis of these three reference systems, with the direction from A to B as the positive direction of this axis.

Now, let A's Einstein-measure of B's relative velocity be denoted by v. For A to determine v, A must measure the distance s_A of the space interval $\overline{AB}$ at two separate times. For one of these times he can use $s_A = 0$ when $t_A = 0$. For the second time, he has to use a reflected light signal by which he obtains B's coordinate in $I_A(x_{AB}, t_A^m)$, where $t_A^m = (\tfrac{1}{2})\,(t_A^1 + t_A^2)$ is the Einstein time measure (8.13) of the reflected light signal, with t_A^1 the time the signal is emitted from A and t_A^2 when the reflected signal arrives back at A. These measures are related to the corresponding I_s measures, s and t_s^r, obtained from (8.17), (8.20) and (8.21) for $\psi = 0$, so that

$$\begin{aligned} t_A^r - t_A^m &= \gamma_A\, s\, v_A/c^2\,, \\ x_{AB} &= \gamma_A\, s\,, \end{aligned} \tag{8.22}$$

where

$$s = (v_B - v_A)\, t_s^r = (v_B - v_A)\,\gamma_A\, t_A^r\,, \tag{8.23}$$

hence

$$v = \frac{x_{AB}}{t_A^m} = \frac{\gamma_A^2\, t_A^r (v_B - v_A)}{t_A^r - \gamma_A^2\, t_A^r (v_B - v_A)\, v_A/c^2} = \frac{v_B - v_A}{1 - v_B\, v_A/c^2}\,, \tag{8.24}$$

which is Einstein's addition theorem of velocities.

Next, consider an event on a body E in arbitrary motion in I_s. Within the plane made up from A, B, and E, one can choose the y-axis perpendicular to the x-axis and the z-axis perpendicular to the plane. The I_s space and time coordinates of the event E are denoted by (x_s, y_s, z_s, t_s^r), where t_s^r is identical to the Einstein time-measure t_s^m by any observer S. A's I_A-coordinates based on A's Einstein-measure of the event are (x_A, y_A, z_A, t_A^m) and those of B's made in the I_B frame are (x_B, y_B, z_B, t_B^m). Therefore, if r is the I_s space interval separating A and E at t_s^r, ψ the angle measured by S that $\overline{AE}$ makes with the x-axis, and if r_A, t_A^m and ψ_A are the corresponding measures according to A, then for the I_s- and I_A-systems one has

$$\begin{aligned} x_s &= v_A\, t_s^r + r\cos\psi, \quad y_s = r\sin\psi, \quad z_s = 0, \\ x_A &= r_A\cos\psi_A, \quad y_A = r_A\sin\psi_A, \quad z_A = 0, \\ &= \gamma_A\, r\cos\psi = r\sin\psi, \end{aligned} \tag{8.25}$$

hence

$$x_A = \gamma_A(x_s - v_A\, t_s^r), \quad y_A = y_s, \quad z_A = z_s. \tag{8.26}$$

Because of (8.20) and (8.21) one then also has

$$\begin{aligned} t_A^m &= t_A^r - (\gamma_A\, v_A/c^2)\, r\cos\psi \\ &= \gamma_A\,(t_s^r - v_A\, x_s/c^2). \end{aligned} \tag{8.27}$$

In a similar way, by relating the I_B- and I_s-measurements, one obtains

$$x_B = \gamma_B(x_s - v_B\, t_s^r), \quad y_B = y_s, \quad z_B = z_s \tag{8.28}$$

and

$$t_B^m = \gamma_B(t_s^r - v_B\, x_s/c^2). \tag{8.29}$$

Eliminating x_s and t_s^r from (8.26–8.29) and using (8.24), one obtains

$$\begin{aligned} x_A &= \gamma_{AB}(x_B + v\, t_B^m), \quad y_A = y_B, \quad z_A = z_B, \\ t_A^m &= \gamma_{AB}(t_B^m + v\, x_B/c^2), \\ v &= \frac{v_B - v_A}{1 - v_B\, v_A/c^2}, \quad \gamma_{AB} = (1 - v^2/c^2)^{-1/2}. \end{aligned} \tag{8.30}$$

The first of these two equations are just the Lorentz transformations with their relative velocity v related to their absolute velocities v_A, v_B in I_s by Einstein's velocity addition theorem. Accordingly, Lorentz invariance can be interpreted as a dynamic symmetry derived from the Lorentz contraction as a true physical effect of a body in absolute motion.

9. On the Meaning of Quantum Gravity

The theory presented supports Einstein's conjecture that all interactions are somehow a manifestation of gravity. It also supports Einstein in his insistence that quantum mechanics is incomplete and that it should be derived from a more fundamental causal structure. But it also supports Newton in his belief that at its most fundamental level nature is atomistic and not a field. Einstein's belief that all fields can be reduced to a noneuclidean space-time structure would be only true in the limit of low energies.

From the explanation of gravitational waves as tensorial vortex lattice-waves, and from the coupling of the tensor h_{ik} describing this wave to the energy-momentum-tensor θ_{ik} one obtains the nonlinear gravitational field equations in flat space-time ($\varkappa = 8\pi\, G/c^4$)

$$\Box\, h_{ik} = \varkappa\, \theta_{ik}, \tag{9.1}$$

where

$$\theta_{ik} = T_{ik} + t_{ik} \tag{9.2}$$

is the sum of the energy-momentum-tensor T_{ik} of matter and the energy-momentum-tensor t_{ik} of the gravitational field h_{ik}. As it was shown by Gupta [12], combining t_{ik} with the l.h.s. of (9.1), and by employing a certain gauge, (9.1) can be brought into the form of Einstein's nonlinear gravitational field equations

$$R_{ik} - \frac{1}{2}\, g_{ik}\, R = \varkappa\, T_{ik}. \tag{9.3}$$

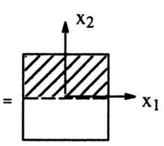
In the form (9.3), the gravitational field can be interpreted as a curved space-time manifold. However, because (9.1) is only valid for small amplitude tensorial vortex waves, large amplitude vortex waves would lead to a departure from (9.1) and hence from (9.3), involving higher order nonlinearities which may not be incorporated into a noneuclidean space-time manifold. These higher order nonlinearities are suppressed by the Planck length. In most cases of astrophysical interest they can be ignored, but they cannot be ignored for the problem of quantum gravity. Superficially it seems this would make the problem of quantum gravity more difficult than it is already with Einstein's equations. In reality they may make the problem of quantum gravity solvable, at least in principle. Because if the gravitational interaction must be viewed as an interaction transmitted through vortex waves in euclidean three-space rather than by a four-dimensional noneuclidean quantized space-time, the problem of quantum gravity is reduced to a problem of nonrelativistic quantum hydrodynamics. And if this hydrodynamics must be understood as a collective description of a very large number of locally interacting Planck mass particles, the problem of quantum gravity is reduced to the solution of a nonrelativistic many body Schrödinger equation. With an equal number of positive and negative Planck masses, the cosmological constant is equal to zero. With all charges reduced to the zero point oscillations of the Planck masses bound in vortex filaments, the sum of all charges, like the electric or color charge, must vanish as well.

Unlike Einstein's gravitational field equations Maxwell's equations are linear, but if they are also to be understood as the result of small amplitude vortex waves, large amplitude waves would there as well lead to nonlinearities for low energies suppressed by the Planck length.

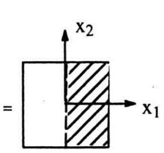
10. Interpretational Problems

One difficult problem in the interpretation of quantum mechanics is the many-body Schrödinger equation in configuration space, leading to the strange phenomenon of phase entanglement. The proposed theory can avoid this problem because it views all particles as quasiparticles of the Planck aether. It there is incorrect to visualize a many-body wave function to be composed of the same particles which are observed before an interaction between the particles is turned on. In the presence of an interaction it rather leads to a new set of quasiparticles into which the wave function can be factorized. This can be demonstrated for two identical particles moving in a harmonic oscillator well. The well shall have its coordinate origin at $x=0$, with the first particle having the coordinate x_1 and the second one the coordinate x_2. Considering two oscillator wave functions $\psi_0(x)$ and $\psi_1(x)$, with ψ_0 having no and $\psi_1(x)$ having one node, there are two two-particle wave functions:

$$\psi(x_1, x_2)=\psi_0(x_1)\,\psi_1(x_2) = \sqrt{\frac{2}{\pi}}\, x_2\, e^{-(x_1^2+x_2^2)/2} =$$

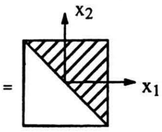


$$\psi(x_1, x_2)=\psi_1(x_1)\,\psi_0(x_2) = \sqrt{\frac{2}{\pi}}\, x_1\, e^{-(x_1^2+x_2^2)/2} = \tag{10.1}$$

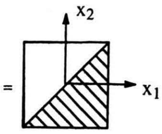


graphically displayed in the x_1, x_2 configuration space, with the nodes along the lines $x_2=0$ and $x_1=0$. By a linear superposition of these wave functions we get a symmetric and an antisymmetric combination:

$$\psi_s(x_1, x_2)=\frac{1}{\sqrt{2}}[\psi_0(x_1)\,\psi_1(x_2) + \psi_1(x_1)\,\psi_0(x_2)] = \frac{1}{\sqrt{\pi}}(x_2+x_1)\, e^{-(x_1^2+x_2^2)/2}$$



$$\psi_a(x_1, x_2)=\frac{1}{\sqrt{2}}[\psi_0(x_1)\,\psi_1(x_2) - \psi_1(x_1)\,\psi_0(x_2)] = \frac{1}{\sqrt{\pi}}(x_2-x_1)\, e^{-(x_1^2+x_2^2)/2} \tag{10.2}$$



If a perturbation is applied whereby the two particles slightly attract each other, the degeneracy for the two wave-functions is removed, with the symmetric wave function leading a lower energy eigenvalue. For a repulsive force between the particles the reverse is true. As regards to the wave functions (10.1), one may still think of them in terms of two particles, because the wave functions can be factorized, with the quantum potential becoming a sum of two independent terms:

$$-\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\psi^*\psi}}{\sqrt{\psi^*\psi}} = -\frac{\hbar^2}{2m}\frac{1}{\sqrt{\psi_1^*\psi_1}}\frac{\partial^2\sqrt{\psi_1^*\psi_1}}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{1}{\sqrt{\psi_2^*\psi_2}}\frac{\partial^2\sqrt{\psi_2^*\psi_2}}{\partial x_2^2}. \tag{10.3}$$

Such a decomposition into parts is not possible for the wave functions (10.2), and it is there then no more possible to think of the two particles which are placed into the well. This, however, is possible by making a 45° rotation in configuration space. Putting

$$y=x_2+x_1, \quad x=x_2-x_1, \tag{10.4}$$

one obtains the factorized wave functions

$$\psi_s=\frac{1}{\sqrt{\pi}}\, y\, e^{-(x^2+y^2)/2},$$

$$\psi_a=\frac{1}{\sqrt{\pi}}\, x\, e^{-(x^2+y^2)/2}, \tag{10.5}$$

for which the quantum potential separates into a sum of two independent terms, one depending only on x and the other one only on y. This means that the addition of a small perturbation in form of an attraction or repulsion between the two particles, transforms them into a new set of two quasiparticles, different from the original particles.

With the identification of all particles as quasiparticles of the Planck aether, the abstract notion of configuration space and inseparability into parts disappears, because any many-body system can, in principle, at each point always be expressed as a factorizable wave function of quasiparticles, where the quasiparticle configuration may change from point to point. This can be shown quite generally. For an N-body system, the potential energy can in each point of configuration space be expanded into a Taylor series

$$U = \sum_{k,l}^{N} a_{kl}\, x_k\, x_l \,. \tag{10.6}$$

Together with the kinetic energy

$$T = \sum_{l}^{N} \frac{m_l}{2}\, \dot{x}_l^2 \,, \tag{10.7}$$

one obtains the Hamilton function $H = T + U$ and from there the many-body Schrödinger equation. Introducing the variables $\sqrt{m_l}\, x_l = y_l$, one has

$$T = \sum_{l}^{N} \frac{1}{2}\, \dot{y}_l^2 \,, \quad U = \sum_{k,l}^{N} b_{kl}\, y_k\, y_l \tag{10.8}$$

which by a principal axis transformation of U becomes

$$T = \sum_{l} \frac{1}{2}\, \dot{z}_l^2 \,, \quad U = \sum_{l} \frac{\omega_l^2}{2}\, z_l^2 \,. \tag{10.9}$$

Unlike the Schrödinger equation with the potential (10.6), the Schrödinger equation with the potential (10.9) leads to a completely factorizable wave function, with a sum of quantum potentials each depending only on one quasiparticle coordinate. The transformation from (10.8) to (10.9) is used in classical mechanics to obtain the normal modes for a system of coupled oscillators. The quasiparticles into which the many-body wave function can be factorized are then simply the quantized normal modes of the corresponding classical system.

For the particular example of two particles placed in a harmonic oscillator well, the normal modes of the classical mechanical system are those where the particles either move in phase or out of phase by 180°. In quantum mechanics, the first mode corresponds to the symmetric, the second one to the antisymmetric wave function. It is clear that the quasiparticles representing the symmetric and antisymmetric mode cannot be localized at the position of the particles placed into the well.

The decomposition of a many-body wave function into a factorizable set of quasiparticles, can in the course of an interaction continuously change, but it can also abruptly change if the interaction is strong. In the latter case, one has what it is known as the collapse of the wave function. Whereas the many-body wave function in configuration space has in terms of a factorizable wave function of quasiparticles a simple interpretation, an explanation of the wave function collapse is much more difficult.

According to von Neumann, quantum mechanics consists of two quite different procedures: 1) a deterministic evolution of the wave function by Schrödinger's equation; and 2) an indeterministic process whereby through a measurement the wave function "collapses" with superluminal speed into one of many alternatives, with the probability for one of the alternatives actually to occur expressed by the wave function prior to the measurement. In the Copenhagen interpretation, the wave function has no real physical meaning, being rather the expression of our knowledge, and it is argued, as our knowledge can change discontinuously following a measurement, so can the wave function. Even though a measurement can always be carried out by an instrument, the Copenhagen interpretation ultimately requires the existence of conscious observers, introducing a highly subjective element into the description of nature. With few places in the physical universe having conscious observers present, the Copenhagen interpretation has not been accepted by all physicists.

In a Newtonian interpretation of quantum mechanics, not only would the Schrödinger equation have to be understood mechanistically, but also superluminal wave function collapse. One may wonder if superluminal wave function collapse might not be inviolation of special relativity, but there are two reasons why this is really not the case. First, as Ehrenfest has shown, a wave packet under the influence of an external force behaves like a particle in classical mechanics. Accordingly, as long as the center of mass of the wave packet does not assume superluminal velocities there is no reason against an internal superluminal motion within the wave packet. It is only this kind of superluminal motion which is required for wave function collapse. Second, because the Planck aether has all the characteristics of a medium, it can have wave modes with divergent phase velocities which may be associated with the superluminal wave function collapse. As long as these modes do not transmit a signal, there can be no violation of special relativity.

In a provisional attempt to show how superluminal wave function collapse may perhaps be understood as a mechanical effect of the Planck aether, we choose the Hartree approximation. It is the most simple approximation of (3.3) in which the field operators $\psi_\pm$, $\psi^\dagger_\pm$ are replaced by their expectation values $\phi_\pm = \langle\psi_\pm\rangle$, $\phi^*_\pm = \langle\psi^\dagger_\pm\rangle$, obtaining the nonlinear Schrödinger equation

$$i\hbar\frac{\partial\phi_\pm}{\partial t} = \mp\frac{\hbar^2}{2m_p}\nabla^2\phi_\pm \pm 2\hbar c\, r_p^2(\phi^*_\pm\,\phi_\pm - \phi^*_\mp\,\phi_\mp)\,\phi_\pm\,. \tag{10.10}$$

By the Madelung transformation it becomes

$$\frac{\partial \boldsymbol{V}_\pm}{\partial t} + \nabla\left(\frac{V_\pm^2}{2}\right) = -2c^2 r_p^3\,\nabla(n_\pm - n_\mp) + \frac{1}{m_p}\nabla Q_\pm\,,$$

$$\frac{\partial n_\pm}{\partial t} + \nabla\cdot(n_\pm \boldsymbol{V}_\pm) = 0\,,$$

$$Q_\pm = \frac{\hbar^2}{2m_p}\frac{\nabla^2\sqrt{n_\pm}}{\sqrt{n_\pm}}\,, \tag{10.11}$$

For small amplitude disturbances with wave lengths large compared to the Planck length, one can neglect the quantum potential and obtains from (10.11)

$$\frac{\partial}{\partial t}(\boldsymbol{V}_+ + \boldsymbol{V}_-) = 0\,,$$

$$\frac{\partial}{\partial t}(\boldsymbol{V}_+ - \boldsymbol{V}_-) = -4c^2 r_p^3\,\nabla(n'_+ - n'_-)\,,$$

$$\frac{\partial n'_\pm}{\partial t} + \frac{1}{2r_p^3}\nabla\cdot\boldsymbol{V}_\pm = 0\,, \tag{10.12}$$

where $n_\pm = 1/2\, r_p^3 + n'_\pm$. Eliminating $n'_\pm$ from the second and third equation of (10.12) one obtains the wave equation

$$\frac{\partial^2}{\partial t^2}(\boldsymbol{V}_+ - \boldsymbol{V}_-) = 2c^2\nabla^2(\boldsymbol{V}_+ - \boldsymbol{V}_-) \tag{10.13}$$

with the dispersion relation

$$\omega^2 = 2c^2 k^2\,. \tag{10.14}$$

For oscillatory disturbances the first equation of (10.12) implies that $\boldsymbol{V}_- = -\boldsymbol{V}_+$ and hence $n'_- = -n'_+$, whereby the total number density of the positive and negative Planck mass particles remains unchanged. Accordingly, the wave does not carry any energy and is "empty".

Next we must consider the coupling of these disturbances with a particle described by the Schrödinger wave function. We first consider the interaction with the Schrödinger wave for a Planck mass. To be described by a Schrödinger equation, it must be distinct from the Planck mass particles of the Planck aether. This is true for a Planck mass bound in a quantized vortex filament, with the diameter of the filament equal a Planck length. Being bound in the vortex filament, the Planck mass executes zero point oscillations determined by the uncertainty principle. This zero point energy is $\hbar c/r_p$ and it generates a virtual phonon field surrounding the Planck mass with the strength of this field equal the strength of the scalar Newtonian gravitational field of a Planck mass. Unlike the better Hartree-Fock approximation, the Hartree approximation does not lead to quantized vortex solutions in the positive-negative mass Planck aether, but the Hartree approximation has the phonon-roton spectrum of a superfluid, and a Planck mass bound in a roton would qualitatively behave like one bound in a vortex filament. Therefore, to make the analysis as simple as possible, we can use the Hartree approximation.

Assuming that all Planck mass particles belonging to the disturbances $n'_\pm$ are bound in rotons, one obtains the following set of small amplitude equations, with the Planck mass particles bound in rotons generating a scalar gravitational potential Φ:

$$\frac{\partial \boldsymbol{V}_\pm}{\partial t} = -2c^2 r_p^3\,\nabla(n'_\pm - n'_\mp) - \nabla\Phi\,,$$

$$\frac{\partial n'_\pm}{\partial t} + \frac{1}{2r_p^3}\nabla\cdot\boldsymbol{V}_\pm = 0\,,$$

$$\nabla^2\Phi = 4\pi G\, m_p(n'_+ - n'_-)\,, \tag{10.15}$$

where we have as before neglected the quantum potential. With $G m_p^2 = \hbar c$, and the second equation of (10.15), one obtains for the gravitational potential Φ

$$\nabla^2\frac{\partial\Phi}{\partial t} = -2\pi\omega_p^2\,\nabla\cdot(\boldsymbol{V}_+ - \boldsymbol{V}_-)\,. \tag{10.16}$$

From (10.15) and (10.16) one then obtains

$$\frac{\partial^2}{\partial t^2}(V_+ + V_-) = 4\pi\,\omega_p^2 (V_+ - V_-),$$

$$\frac{\partial^2}{\partial t^2}(V_+ - V_-) = 2c^2\,\nabla^2 (V_+ - V_-). \qquad (10.17)$$

As before, the second equation of (10.17) has wave-like disturbances obeying the dispersion relation (10.14), but it has in addition also the special solution $V_+ - V_- = A = \text{const}$. Inserting this special solution into the first of (10.17) one obtains for $(V_+ + V_-)$ a solution rising in time:

$$(V_+ + V_-) = 2\pi\,\omega_p^2 (V_+ - V_-)\,t^2. \qquad (10.18)$$

For $V_- = 0$, with $\omega_p^2 = 2G\,n_+\,m_p = 2G\varrho$, (10.18) becomes

$$t = 1/\sqrt{4\pi G\varrho}, \qquad (10.19)$$

which is the gravitational collapse time for a mass of density ϱ. If $V_- \to -V_+$, by which the second equation of (10.17) approaches the "empty" wave solution, one has $t \to 0$. The gravitational collapse time can for this reason be substantially shortened in the presence of negative masses, if the negative masses flow is in a direction opposite to the flow of the positive masses. Because the shortening of the collapse time occurs when the net average density approaches zero, as it is the case for the "empty" wave, we suggest that this kind of gravitational collapse may serve as a model for wave function collapse.

Assuming that the ratio $\hbar\omega/\hbar\omega_p$ of the kinetic energy of the Planck mass described by Schrödinger's equation to the Planck energy is equal to $(|V_+|^2 - |V_-|^2)/(|V_+|^2 + |V_-|^2)$ we can set near $V_- \simeq -V_+$, $\omega/\omega_p \simeq (|V_+| + |V_-|)/|V_+|$, hence $(V_+ + V_-)/(V_+ - V_-) = (1/2)\,\omega/\omega_p$. We thus find for (10.18)

$$t^2 = \frac{\omega}{4\pi\,\omega_p^3}. \qquad (10.20)$$

Because the time for the collapse should be of the order $t \sim 1/\omega$ one finally has

$$t \simeq (4\pi)^{-1/3}\,r_p/c. \qquad (10.21)$$

A wave packet of width r of a Planck mass described by Schrödinger's equation, would collapse with the superluminal speed

$$v_c = r/t \sim (r/r_p)\,c. \qquad (10.22)$$

In generalizing this result to a Schrödinger equation describing a mass $m < m_p$, we have to replace in (10.12) and (10.15) r_p with $r_0 = \hbar/mc$, and find instead of (10.21)

$$t \simeq (4\pi)^{-1/3}\,r_0/c \qquad (10.23)$$

with (10.13) remaining unchanged. For the collapse velocity we obtain instead of (10.22)

$$v_c \sim (r/r_0)\,c. \qquad (10.24)$$

It is plausible that wave-function collapse should have a higher probability to occur towards a region of the wave packet where $\psi^*\psi$ is large.

For the collapse to proceed along the lines suggested by the model, the wave-like disturbances of the Planck aether must be in phase. With the Planck aether likely to be subject to large scale fluctuations, possibly rising in proportion to $r^{1/3}$ as for a turbulent fluid, the mechanism for the collapse may not work above a certain length. If this should turn out to be true, then the quantum mechanical correlations are going to break down above this length.

The Planck aether defines an absolute system needed if one wishes to explain the long-range quantum mechanical correlations and the wave function collapse as real physical phenomena. Otherwise, the time sequence of cause and effect in the measurement of two correlated events can be interchanged by a Lorentz transformation to another reference system. The existence of a preferred reference system is also plausible from the cosmological evidence. It does not show us a system of galaxies with a large velocity dispersion (like the molecules in a gas), something which would still be in accordance with the postulates of special relativity. It rather shows us an ordered uniformly expanding system, suggesting that all the matter in the universe comes from bound states of a fundamental field at absolute rest in the system of galaxies.

11. Concluding Remarks

According to Weinberg [13], the Lagrange density for the standard model suppresses higher order terms by the Planck mass (resp. Planck length). The standard model must for this reason be viewed as an asymptotic low energy limit. Here we have shown that the same may be said not only about Lorentz invariance, but even about quantum mechanics. Under a few assumptions, Lorentz invariance as well as quantum mechanics can be viewed as asymptotic approximations for energies small compared with the Planck energy. Postulating the existence of positive as well as negative masses, the proposed fundamental law is reduced to nonrelativistic Newtonian mechanics with short range interactions. Newton's ultimate objects, identified with the Planck mass particles, resemble Leibniz's monads. Like Newton's ultimate objects, Leibniz's monads are indestructible. The property of the monads to have no windows is reflected in the property of the Planck mass particles that they are not the source of long-range forces. Identifying the Planck mass particles with these indestructible elements requires that they obey an exactly nonrelativistic law of motion because only in a nonrelativistic quantum field theory does the Hamilton operator commute with the particle number operator. And only under this restriction was it possible to derive quantum mechanics and special relativity as low energy approximations. Finally, in the proposed theory the quantum mechanical uncertainty is not fundamental but rather hidden within regions smaller than the Planck length, due to the impossibility to make any measurement for distances smaller than this length. What happens within this region determines the individual outcome of the "Zitterbewegung" motion for each particle, but it is uncertain for an observer who cannot look down into regions smaller than r_p. This uncertainty is in full agreement with Heisenberg's uncertainty equation at the Planck length $m_p r_p c = \hbar$.

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